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Dynamics of Hand-Held Impact Weapons

Sive De Motu

George L Turner
Association of Renaissance Martial Arts
http://www.thehaca.com

Foreword

Hand weapons are no longer well understood from a theoretical standpoint. Actually, it seems they are no longer understood at all. For all the talk about Rockwell hardness and charts of percussion point locations, one might think we know more about swords now than ever. This is blatantly not true. Despite hundreds of years of fencing, and scores of people studing the martial arts of the East, we almost certainly know less than the average knight did, and probably the average Greek hoplite. After the Atlanta 2001 conference, some talks with John Clements nagged at me, and I realized that there are some basic assumptions about the weapon, which have been passed down to us from prevoius writers, that just aren't matching up with what's now coming to light in his research. I searched the web for even basic information on the physics of swords, but from what few sources there are, I couldn't even confirm that swords are swung in rotation. I thought that there would be a great deal of information from either the fencing, Eastern martial arts, or sword making communities, but came away empty handed. Well, actually I did find information, but most of it was either very simple or just plain wrong. Even the simple certainties were wrong, such as the sword's center of percussion always being about a third back from the tip, which has turned out to be completely false. So, in frustration, I've had to figure out some weapon theory myself, even though I know but very little about swords or physics. So I'm no expert at this stuff, but from the looks of things no one else has seriously bothered with it either, at least since the early 1700's. Interestingly, the only post 1600's tool that I needed was just the concept of conservation of energy. Any competent 17th century researcher could've done everything else in this work, and apparently did so, if the Philosphical Transactions of the Royal Society, circa 1660-1670, are any indication. So I've been bouncing ideas off John Clements, Matt Hauser, Richard Boswell, and G. Wade Johnson, all with ARMA, and here's the result. Few of the results were anticipated, and the research has been a constant eye opener. I'm very irritated that someone hadn't done this kind of work some time during the past two hundred years. As it requires nothing but the simplest Newtonian mechanics, or even pre-Newtonian mechanics, I can think of no excuse for neglecting it, other than most people being content with pat answers to complex questions. It is becoming obvious that if a modern reenactor were transported back to the 1600's, both his sword and his explainations of it would meet with derisive laughter.

But if you think the following are true, then this essay will present a startling different view of the sword.

- Weight and balance point are the primary determinants of a sword's feel.
- The pommel is primarily used to balance the sword, setting the location for the center of mass.
- The percussion point is well determined by striking the side of the blade, and is always about 1/3 of the blade length back from the tip.
- Strikes with the percussion point leave the least vibration in the blade.
- Strikes with the percussion point do the maximum damage, and leave no energy in the blade.
- When struck, a sword will rotate around its center of mass.
- The cross guard is primarily to protect your fingers.
- Hand shock drains energy from the blow.
- Heavier swords stike harder than light swords.

This research makes use of simple high school algebra, but many of the most startling results require no math at all. After all, back then most swordsmen weren't mathematicians, either. If you hate math please feel free to skip the math sections and look for the red underlined passages that highlight the findings. All the physics will be in the metric system just to keep the units simple. I will, however, also give a measure called "the moment of inertia" in English units, as it's as fundamental to a sword as mass is, and can be used to communicate information about a sword's feel over the web. In English units, the moment of inertia also matches up pretty well with the sword's weight, which makes it easy to intuitively get an idea of how a sword will behave from just a verbal description.

At several points in this article, I will bring up some commonly held but invalid beliefs that are circulating in some parts of the sword community. This is done to examine preconceived, but invalid, notions in detail. Unless they are closely scrutinized they will continue to circulate in the back of your mind, popping up to provide a misunderstanding of what you are both seeing and feeling in your practice. I will also bring up some other intuitive ideas that don't necessarily match up with reality, but might bring some insight into what ancient smiths and swordsmen may have *thought* was happening in a blow. Then again, they may have truly known what happens, since they had so much empircal information to work with.

Notation used in this article

Symbol	Meaning	Units
m	mass	kg
COM	center of mass	
I	moment of inertia	$kg\text{-}m^2$
$\mathbf{I_{COM}}$	moment of inertia calculated around the center of mass	$kg\text{-}m^2$
$\mathbf{I_X}$	moment of inertia calculated around the point at **x**	$kg\text{-}m^2$
H	angular momentum	$kg\text{-}m^2/sec$
F	force	newtons (newton = $1kg\text{-}m/sec^2$)
Γ	gamma, torque	newton meters (N-m)
a	acceleration	m/sec^2
v	velocity	m/sec
s	distance	m
α	alpha, angular acceleration	$radians/sec^2$
ω	omega, angular velocity	radians/sec
θ	theta, angle	radians

Chapter 1
Moment of Inertia

A sword has many physical properties that can't be fudged, such as mass. If your replica weighs significantly more or less than the original, it will not handle the same. A heavier sword is harder to accelerate and decelerate in thrusting. Another physical property is the location of the center of mass (COM). When multiplied by the sword's mass this gives the moment of force required from your hand to counter accelerations perpendicular to the blade, including gravity. If you have a pair of 3-pound swords where one has twice the distance to the balance point as the other, it will take twice as much torque to keep it pointed straight ahead when you shift it side to side. It will also take twice the torque from your forearms to hold it level against the force of gravity.

A third parameter is called the moment of inertia (MOI), a more complicated, yet still fundamental measure. The moment of inertia is the rotational equivalent of mass. It measures how hard it is to rotate the sword, and is determined by how mass is distributed throughout the sword and the axis about which the sword is rotated. If the mass is spread too far towards the tip and pommel it will rotate like a barbell. Even if your sword has the same weight and balance point as an original, it may be very far off in moment of inertia. Mass and balance point location are not sufficient to characterize the feel of a hand weapon. Unfortunately, unlike mass or moment of force, the inertia has a square law property. It is the sum of each bit of mass multiplied by the distance to the mass SQUARED. The moment of inertia can change very quickly with only small changes in the sword's weight or balance point.

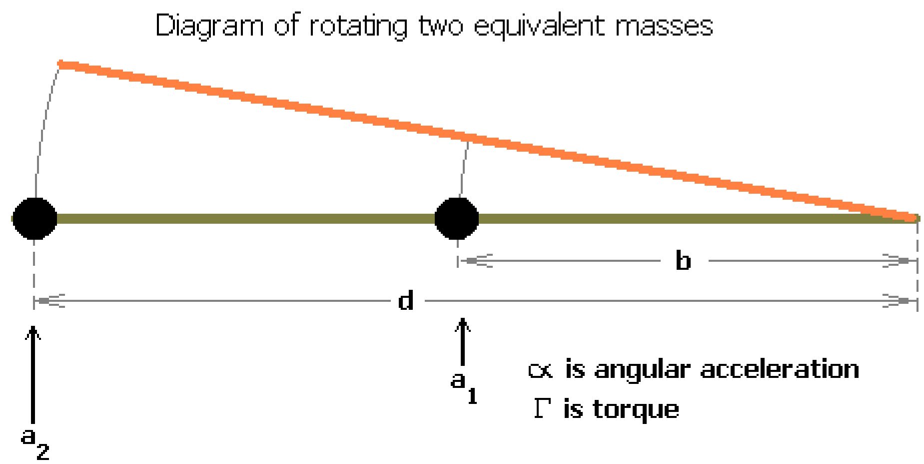
Your sword can have the exact weight and balance point as an original, but the little details like tip thickness can devastate its performance. If the last foot of your blade is 1/16" thicker than an authentic sword, it can increase the moment of inertia by 50%. Static balancing only checks the COM, not the MOI. Existing tests, other than just grabbing the sword and waving it around, can't detect these flaws. Some of you have probably handed John Clements a sword that had authentic weight and balance point, yet he waves it around and pronounces some serious handling flaw. He's not only checking the grip and hand interface, he's instinctively checking the moment of inertia. As far as the current sword community goes, he's performing a "beyond the state of the art" test. You can't compensate for these errors by adding pommel weight, as this just adds to the moment of inertia. Adding mass anywhere on a weapon will always increase the moment of inertia calculated about any axis. The reason being that no matter where you grip, you have to swing the original weapon, plus the added mass. The only way to reduce it is to remove mass, and the further the distance from your hand to the removed mass, the better - squared.

> Note: You may have heard quite a bit about a sword's moment of inertia in regards to its blade cross section. This is quite unrelated to the moment of inertia that we're concerned with here. It so happens, that to calculate the cross sectional stiffness of an object, you end up with an $\mathbf{mx^2}$ calculation, where $\mathbf{x}$ is the distance from the neutral axis of the beam. This happens to be the moment of inertia of an infinitely thin cross section of the beam, and is quite useful in designing for stiffness. But it is not quite the same use we're talking about when we discuss swinging a blade, as opposed to bending it.

Derivation of Mass Moment of Inertia

The moment of inertia is easily calculated, but a bit harder to intuitively understand. High school physics books don't give it a great deal of attention, except to introduce it as an interesting concept that lets you calculate the final velocity of a wheel rolling down an inclined plane. The concept of mass moment of inertia is quite simple. Note that mass moment of inertia isn't a typo, it's a descriptive phrase used to distinguish the term from area moment of inertia, which is a related, but slightly different concept.

> **The Why of the Square Law**
>
> For the mathematically inclined, here's a basic diagram of why the moment of inertia obeys a square law. It all boils down to the fact that if the mass is twice as far away, it must experience twice the tangential acceleration for the same angular acceleration. So this doubled acceleration must take twice the force, and this force must be applied with twice the moment arm. Twice as fast times twice as far gives four times the torque. Three times as far means three times the force with three times the moment arm, or nine times the torque.



tangential acceleration *mass #1*

$a_1 = b \propto$

required force

$F_1 = m\,a_1$

required torque

$\Gamma_1 = F_1\, b$

tangential acceleration *mass #2*

$a_2 = d \propto$

$a_2 = \frac{d}{b} a_1$

required force

$F_2 = m\,a_2$

$F_2 = m \frac{d}{b} a_1$

required torque

$\Gamma_2 = F_2\, d$

ratio of forces for same ∝

$F_2 = \frac{d}{b} F_1$

$\Gamma_2 = \frac{d}{b} F_1\, d \qquad F_1 = \frac{\Gamma_1}{b}$

ratio of torques for same ∝

$\Gamma_2 = \frac{d^2}{b^2} \Gamma_1$

$$\boxed{\Gamma_2 = \left(\frac{d}{b}\right)^2 \Gamma_1}$$

Here, for a given angular acceleration α (alpha), the tangential acceleration, a, of each mass, is determined by the distance of the mass from the axis of rotation. The ratio of the linear acceleration of the second mass, compared to the first, is the same as the second masses distance from the axis compared to the first masses (d/b). But it also takes more torque to move the farther mass, even at the same linear rate. This torque ratio is also d/b, which has to be multiplied by the increased force you have to apply, because the distant mass has to move farther to cover the same angle. The Greek letter gamma (Γ) is a torque, or moment of force. A handy way to remember the meaning of gamma is to picture a crow bar, which is used to apply a torque to something. You don't need to keep this math in your head. Just remember that moment of inertia follows a square law.

In terms of static balancing, if you add 1 kg at 1 meter from the hilt, it's the same as adding 2 kg at ½ meter from the hilt. But for moment of inertia adding 1 kg at 1 meter is the same as adding 4 kg at ½ meter. Dynamic weight isn't the same as static weight! If you take a large barbell and slide the weights to the middle, the center of mass and total weight are unaffected. But it will rotate very differently!

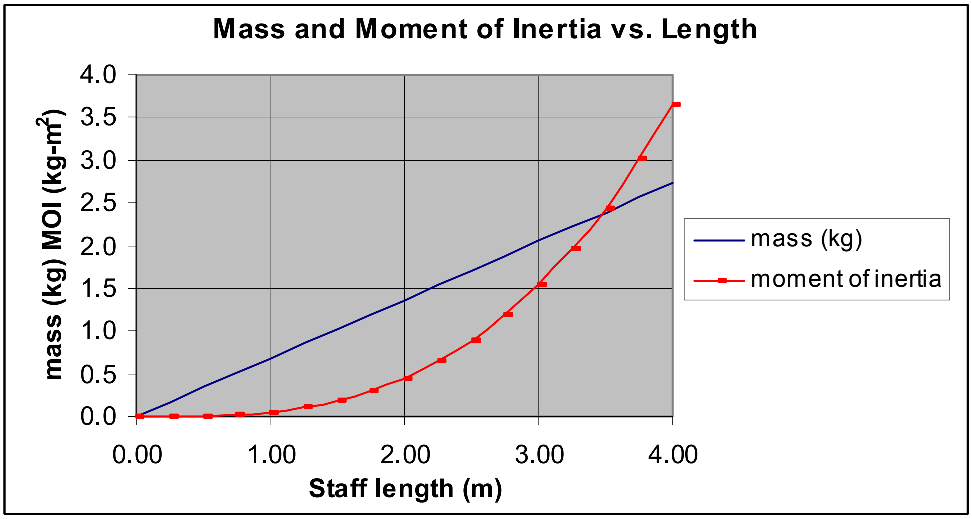
A Bit of Math

The moment of inertia is usually represented by the letter **I**. It is always calculated relative to some axis, as it consists of the sum of all the little microscopic masses times their distance from the axis squared. In short

$$\mathbf{I}=\sum_{i=1}^{n} m_i x_i^2$$

where $\mathbf{x_i}$ is the distance from the axis to the $\mathbf{i}^{th}$ mass. So if you've figured out the moment of inertia of an existing object about some particular axis and then add some new mass, calculate the moment of inertia of the new mass about the particular axis, then just add it to the old moment of inertia. After all, you now have to swing exactly the old object of known moment of inertia plus the new object's moment of inertia. The new mass just gets added into the above equation as $\mathbf{m_{n+1}x_{n+1}^2}$. This is handy when you look at adding new components to a weapon, or trying to break an existing weapon into constituent components.

To really encounter what moment of inertia is, go to a lumberyard. Pick up a piece of lumber that's four feet in length and swing it around by the middle. Then go pick up a 16-footer and try the same. It weighs four times more, so translations and thrusts are four times slower. But what is the difference in swing weight between the two boards? Is it four times worse, because the bigger board weighs four times more? No, is it sixteen times worse, because it's four times longer and four times heavier? No, it's sixty four times worse, because inertia is mass multiplied by length squared. That's how fast the moment of inertia gets out of hand. Four times longer and sixty-four times harder to swing. Any of you whose worked construction knows how hard it is to get a long board rotating, and then how hard it is to stop the rotation. That's the moment of inertia rearing its ugly head, and the same thing shows up in swords. To keep it under control, the tip mass must be kept very small.



This chart shows how increasing length increases the mass and moment of inertia about the center of mass of a 1.5-inch (38mm) diameter staff, made of ash or white oak. Note how fast the moment of inertia builds up, making a long staff very unwieldy. For a given torque applied by your arms, the staff's maneuvering accelerations are inversely proportional to the moment of inertia. So the maximum attainable rotational acceleration is plummeting as length increases.

The Parallel Axis Theorem

The parallel axis theorem is a very handy tool that let's us compute the moment of inertia at any point on the weapon, as long as we know the weapon's moment of inertia about its own center of mass, or balance point. This is double handy, since it also lets us calculate this all important moment of inertia about the center of mass, just by knowing the moment of inertia of another point, and the distance from that point to the center of mass. Given a weapon of mass m, for any point, removed by some distance x from the weapon's center of mass, the all-important moment of inertia at that point is calculated simply as $I_X = I_{COM} + m \cdot x^2$. If you know the moment of inertia at some point on the weapon, you can back calculate the weapon's moment of inertia about its center of mass as $I_{COM} = I_X - m \cdot x^2$. This is simply called the parallel axis theorem, and you have to get familiar with it to do any serious hand weapon calculations.

More on The Parallel Axis Theorem

The distance from your hand to the weapon's center of mass will also increase the weapon's moment of inertia. Due to the nature the physics, the moment of inertia is always smallest when computed around the weapon's center of mass. Every other point involves an increase in moment of inertia. Fortunately there is a handy formula for calculating this increase. This trick is called the parallel axis theorem. It states that when you shift along an axis parallel to the one used to compute the objects moment of inertia about its center of mass ($\mathbf{I_{COM}}$), you merely have to add the mass times distance squared of the move. You don't have to start from scratch in measuring the inertia about the new point. You just add $\mathbf{mx^2}$ where **m** is the weapon's mass and **x** is the distance from the new location to the center of mass, $\mathbf{I_X} = \mathbf{I_{COM}} + \mathbf{mx^2}$. If you've measured the inertia about some arbitrary point that's located at distance **y** from the center of mass and want to figure out the inertia about the center of mass, just use the formula $\mathbf{I_{COM}} = \mathbf{I_Y} - \mathbf{my^2}$. Just heed two simple warnings. You can never get between two arbitrary points with the parallel axis theorem. The math just doesn't work for this. So if you know some $\mathbf{I_Y}$ and want to know an $\mathbf{I_X}$, you must go from $\mathbf{I_Y}$ to $\mathbf{I_{COM}}$ using $\mathbf{I_{COM}} = \mathbf{I_Y} - \mathbf{my^2}$, then go to $\mathbf{I_X}$ with $\mathbf{I_X} = \mathbf{I_{COM}} + \mathbf{mx^2}$. The other warning is that you must use it around an axis parallel to the original axis, so don't do something wacky, like trying to use it to calculate the inertia of rotating the sword like a drill bit. To do this you get into high-level math, inertial dyadics, and such. If you can handle those then you're ready to program space probe maneuvers for NASA.

In the rest of this essay you'll see $\mathbf{I_{COM}} + \mathbf{mx^2}$ all over the place, and this just means the moment of inertia computed about the point at distance **x** from the center of mass.

The Effects of the Moment of Inertia

If the moment of inertia of your sword is large, then it won't point well. It will feel clumsy, and will take a great deal of torque from your hands to move the tip around. Words to describe a sword with a small moment of inertia would be quick, agile, light, maneuverable, and fast. A high moment of inertia brings to mind slow, cumbersome, dead, clumsy, club like, and ponderous. If you pick up a sword with a high moment of inertia it will feel fine until you try to rotate the blade. You might stop, recheck the balance point, and shrug. It just feels dead. It moves slowly, handles poorly, and you keep shopping for another sword.

This is not new to those who study baseball, golf, tennis, or other such sports. The NCAA (U.S. National Collegiate Athletic Association) did a recent study, and has decided that baseball bats shall be regulated based on their moment of inertia, since the moment of inertia, not the mass, was correlating to bat speed. See http://asb-biomech.org/NACOB98/207/. Golfers are also up on this bit of knowledge, as discussed in the following link, http://www.ottawagolf.com/swingsync/swingweight.htm. For tennis players the swing weight and other impact properties are very important (http://tennisone.com/Product/racquet.kp/racquet.custom2.kp.htm), and this is in highly competitive sports, where the feedback on design flaws is immediate. Even croquet has produced some useful data. The detailed physical properties and behavior of sports equipment is considered quite important. Athletes can win and lose because of their equipment. And weaponry? Well, it was people's lives and empires being won and lost. Maybe the historians view that as more important.

So what does a horribly inaccurate moment of inertia do to your technique? It screws it up. Consider the case of two students whose swords have the correct weight and balance, but with moments of inertia double the historical values. They can execute moves in translation, where the sword moves forward and back, or side to side, in the historically correct times. But in pure rotation, their swords accelerate at half the proper rate. Given the same torque applied to the handles, changing blade orientations through pure rotation will take 41% longer than with a historically accurate sword. If they are trying to follow the techniques of some historical master, then techniques that oppose a move in translation with a move in rotation won't work properly. The rotation will take too much time, and they will think they're doing something wrong. They are. They're using a historically inaccurate blade. Unfortunately they'll keep changing their timing and distances until they can make the master's advice "work," or conclude that the particular technique is flawed.

So why not just make the sword have the smallest moment of inertia possible? Why not make it a lightweight super sword? Well, because then it can't deliver any force. If the sword's moment of inertia is decreased to zero, the speed doesn't go up to infinity, because even an empty hand rotates with relatively small, finite speed. The delivered momentum and energy have gone to zero. Such a blade will make the wielder look fast, like a fencer, but thankfully he won't be able to hurt anyone. This would also be making up a new combat style, not learning a historical one.

Handy Formulas

Linear

$F=ma$	Force = mass * acceleration
$v=at$	Velocity = acceleration * time
$s=\frac{1}{2}at^2$	Distance = ½ acceleration * time squared
$L=mv$	Momentum = mass * velocity
$KE=\frac{1}{2}mv^2$	Kinetic energy = ½ mass * velocity squared

Rotational

$\Gamma=I\alpha$	Torque = moment of inertia * angular acceleration
$\omega=\alpha t$	Angular velocity = angular acceleration * time
$\theta=\frac{1}{2}\alpha t^2$	Angle = ½ angular acceleration * time squared
$H=I\omega$	Angular momentum = moment of inertia * angular velocity
$KE=\frac{1}{2}I\omega^2$	Kinetic energy = ½ moment of inertia * angular velocity squared

There are two ways to calculate kinetic energies, etc. One is to calculate the linear velocity of the center of mass (**COM**) and then add in the angular velocity of the weapon, which is computed with the weapon's moment of inertia about the center of mass (I_{COM}). This gives **$KE=\frac{1}{2}m v_{COM}^2 + \frac{1}{2} I_{COM}\omega^2$**. The other method is to use the moment of inertia about the axis of rotation and the linear velocity of the axis of rotation. The results are the same. **$KE=\frac{1}{2}m v_{AXIS}^2 + \frac{1}{2} I_{AXIS}\omega^2$**. Just don't get confused about it.

The inertia is computed about an axis by taking the objects inertia about its center of gravity, and adding the objects mass, multiplied by the distance to the axis squared. This mx^2 comes to dominate the value whenever x gets large. So where the wrists might see the inertia as doubling, at the elbow it's maybe gone up only about 25%, and at the shoulder maybe 20%. And this is ignoring the inertia already in the forearm and upper arm. So your big muscles aren't noticing it much. That big heavy Conan blade can still be swung. It just can't be maneuvered very well. It's an axe (18 lb-ft^2 for a standard, single edged, 3.5-lb head, and swung all day by lumberjacks).

In more detail, let's look at a simple wrist cut through any given angle. Given two swords, which have moments of inertia varying by a factor of two, for the same applied torque the final rotational velocity of the heavier sword is only 71% as fast, but the kinetic energy is unchanged and the momentum is 41% higher. Coming through a given angle, the sword with the higher swing weight will have more momentum, and pay no kinetic energy penalty. Keep in mind that this only directly applies to wrist cuts, however. The physics of real cuts get complicated, and quickly, and to make sure that they, too, are historically valid, do them with a historically valid sword.

So it seems like the choice of proper swing weight is a compromise, no matter what you choose, and it is. But instead of relying on our subjective opinions as to where to make that compromise, we can rely on the purchasing decisions of very knowledgeable and immensely informed consumers. Unfortunately they're long dead, but if you're going to study historical swordsmanship then you're also, indirectly, trying to get to the skill level and mastery that would lead you to make the same purchases that they did. They didn't buy a sword based only on looks. They knew how it had to perform.

This should prevent further frictions in the sword community over feel. No one gets upset when they ask you about their sword and you reply that it weighs 6.5 pounds. They know it, and they know what it's supposed to weigh, or should. But if the problem is swing weight, you have to resort to telling them that their expensive, custom sword feels like crap. It's not just an opinion or an insult. It's just that we've lacked a better way to express this parameter. Now you'll be able to say "Whoa! Its swing weight is 7.5 pound-feet squared! Are you crazy? Historical examples of this type are always 3.5 to 4 pound-feet squared. Paul Bunyon couldn't swing this thing!"

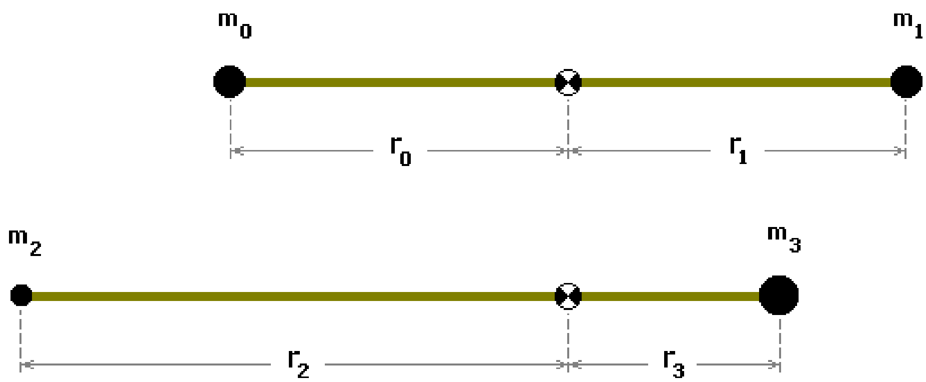
As an aside, a fencing foil has a moment of inertia, computed about the finger and thumb, of around 0.65-lb-ft^2. Replica two-handed swords, even a fine Italian model, can go at least as high as 18-lb-ft^2, while an authentic Swedish true-two handed sword, the five foot long monsters, might come in at around 5.5-lb-ft^2. A typical Viking sword might weigh around 2.1 lbs, and also have a moment of inertia of around 2-lb-ft^2. A single-handed sword from the 1400's weighing around three pounds will probably come in at around 2.5-lb-ft^2.

At the end of this essay is a section on techniques of measuring your sword's moment of inertia, but will leave that for later. There are much more interesting things to pursue right now, so on we go.

Extra, For People with Too Much Time on their Hands

The radius of gyration is another parameter of physical objects. A strange sounding, but simple, term. It merely means that you can make an object of a given mass m and moment of inertia I_O about the axis O, by placing the mass at distance k from the axis O, where $k = \sqrt{\frac{I_O}{m}}$.

The moment of inertia of a point mass m about axis O is given as $I_O = m \cdot k^2$, where k is the distance from the axis of rotation to the point mass m. This distance k is called the radius of gyration, and is handy because it allows us to treat the sword as a point mass, yet get the correct mass and moment of inertia. However, this is ignoring the sword's balance point, and for a general solution it must also be correctly modeled. So let's split the mass m into two small masses, whose sum is equal to the mass m. Then we can match up the mass, moment of inertia, and balance point. Some types of analysis can also be greatly simplified by the simple two mass model of a sword.



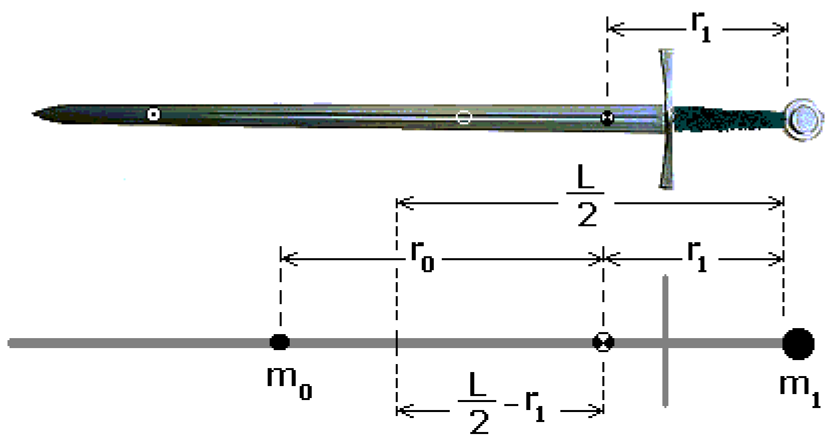
Here, the mass of the each sword is just the sum of the two masses on the end. If we want the same mass and balance point for both swords, then we have $m_{SWORD} = m_0 + m_1 = m_2 + m_3$. But in order to balance, we also know that $m_0 \cdot r_0 = m_1 \cdot r_1$, and $m_2 \cdot r_2 = m_3 \cdot r_3$. The moment of inertia about the center of mass is just the sum of both masses, multiplied by the square of the distance to those masses. This gives us $I_{COM} = m_0 \cdot r_0^2 + m_1 \cdot r_1^2$ for the first sword, and $I_{COM} = m_2 \cdot r_2^2 + m_3 \cdot r_3^2$ for the second sword.

For a given mass, moment of inertia, and distance from the center of mass to the pommel, we can exactly calculate the positions and masses necessary for a solution. So given m_{SWORD}, I_{COM}, and r_1, we must find m_0, m_1, and r_0. First, note that the total mass requires that $m_0 = m_{SWORD} - m_1$, and balance dictates that $r_0 = \frac{m_1 \cdot r_1}{m_{SWORD} - m_1}$. Using these equations to substitute for $m_0 \cdot r_0^2$ in the equation $I_{COM} = m_0 \cdot r_0^2 + m_1 \cdot r_1^2$, gives us an equation with only m_1 as an unknown, which is $I_{COM} = (m_{SWORD} - m_1) \cdot \left(\frac{m_1 \cdot r_1}{m_{SWORD} - m_1} \right)^2 + m_1 \cdot r_1^2$. Solving this for m_1 gives us $m_1 = \dfrac{m_{SWORD}}{\left(\dfrac{m_{SWORD} \cdot r_1^2}{I_{COM}} + 1\right)}$. From this we can easily solve for r_0, using

$$r_0 = \frac{m_1 \cdot r_1}{m_{SWORD} - m_1}.$$

Building a Feel Simulator

Suppose we wanted to simulate a museum sword that has a known mass, center of mass to pommel distance, and moment of inertia about the center of mass. This is exactly the problem we just solved, with a couple added complexities. We must somehow connect the masses, which means we have a rod with some moment of inertia already present. The masses are also no longer point masses, but real weights, which have a moment of inertia of their own. Let's base the simulator on a length of aluminum tubing, which is commonly available at any hardware store. I'll use a 4 foot piece of ¾" aluminum tubing, which weighs 200 grams. By textbook formula, the moment of inertia of a long thin rod is given by $I_{COM\_ROD} = \dfrac{m \cdot L^2}{12}$, so in this case the tube has a moment of inertia of only 0.025 kg-m2, just slightly more than a fencing foil, which weighs 0.5kg and an I_{COM} of about 0.0212 kg-m2, or 0.501 lb-ft2.



First, calculate the moment of inertia of the rod about the desired center of mass location. The distance from the pommel end of the rod to the sword's center of mass is just r_1, as shown, while the distance to the center of the rod is $\dfrac{L}{2}$, where L is 4 feet, or 1.219 meters. So we want to know the moment of inertia of the rod when we shift to distance of $\dfrac{L}{2} - r_1$. Using the parallel axis theorem, this moment of inertia will be $I_{COM\_NEW} = I_{COM\_ROD} + m_{ROD} \cdot \left(\frac{L}{2} - r_1\right)^2$.

This can also be written as $I_{COM\_NEW} = \frac{m \cdot L^2}{3} - m \cdot r_1 \cdot (L - r_1)$. This will be subtracted from the sword's moment of inertia to tell us how much moment of inertia must be added by our weights.

The masses of the two weights must bring the rod's weight up to that of the sword, so we have $m_0 + m_1 + m_{ROD} = m_{SWORD}$. The rod must end up balancing at the same point as the sword, as measured from the pommel, and we already know the rod by itself is tip heavy. The mass times the distance to the left of the sword's center of mass is $m_{ROD} \cdot \left(\frac{L}{2} - r_1\right) \quad + \quad m_0 \cdot r_0$, while to the right we merely have $m_1 \cdot r_1$. So we get $m_{ROD} \cdot \left(\frac{L}{2} - r_1\right) \quad + \quad m_0 \cdot r_0 = m_1 \cdot r_1$.

To simplify things, lets at some mass m_B to make the rod balance at the desired location. This mass, multiplied by the distance r_1, should match the value of $m_{ROD} \cdot \left(\frac{L}{2} - r_1\right)$, so

$$m_B = \left(\frac{m_{ROD} \cdot \left(\frac{L}{2} - r_1\right)}{r_1}\right).$$

So we now have a rod and a balancing mass, which means we merely need to add some additional masses at distances r_0 and r_1 to add to the mass and moment of inertia of the original sword. This is just like the simple case of two pure masses, but in which we already have a partial mass $m_{ROD} + m_B$ and moment of inertia of $I_{COM\_NEW} + m_B \cdot r_1^2$. If we subtract these new values for mass and moment of inertia from the desired final properties of the sword simulator, we've then reduced the problem to exactly the case of the pure two mass system. So we have the rod correctly balanced, but with only part of the mass and moment of inertia required for the final sword. We now need to add our two additional weights, such that we increase the mass and moment of inertia according to the following equations.

$$m_{ADDITIONAL} = m_{SWORD} - (m_{ROD} + m_B)$$

$$I_{ADDITIONAL} = I_{SWORD} - (I_{COM\_NEW} + m_B \cdot r_1^2)$$

Using the equations developed for the simple two-mass system, we have a mass to add to the pommel, at distance r_1, given by $m_1 = \frac{m_{ADDITIONAL}}{\left(\frac{m_{ADDITIONAL} \cdot r_1^2}{I_{ADDITIONAL}} + 1\right)}$. The other mass, located further toward the tip, has a mass given by $m_0 = m_{ADDITIONAL} - m_1$, and is located past the center of mass by a distance given as $r_0 = \dfrac{m_1 \cdot r_1}{m_{ADDITIONAL} - m_1}$.

Minimum and Maximum Obtainable Moment of Inertia

The minimum moment of inertia possible in such a simulator, while still matching a given sword's weight and balance point, is actually obtained by taking all the required additional mass and using it only to balance the sword. If we stay with the term r_1 to represent the desired pommel to center of mass distance, then let's call the new mass m_m and make its distance from the center of mass, r_m as small as is required. So the additional mass is all toward your hand, and is just $m_m = m_{SWORD} - m_{ROD}$. In order to balance, we must make sure that

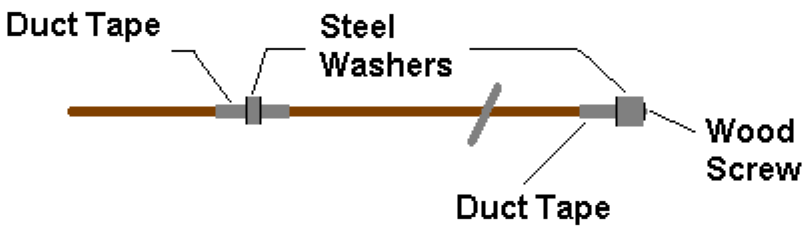
$$m_{ROD} \cdot \left(\frac{L}{2} - r_1 \right) = m_m \cdot r_m$$, or $$r_m = \frac{m_{ROD} \cdot \left(\frac{L}{2} - r_1 \right)}{m_m}$$.

The maximum moment of inertia is obtained by moving as much mass out to the tip as is possible, while still maintaining the desired balance point. The mass will be located at a distance of $r_0 = L - r_1$ from the desired center of mass location. In order to balance, we now have $m_0 \cdot r_0 + m_{ROD} \cdot \left(\frac{L}{2} - r_1 \right) = m_1 \cdot r_1$. Substituting in for r_0 we change this to $m_0 \cdot (L - r_1) + m_{ROD} \cdot \left(\frac{L}{2} - r_1 \right) = m_1 \cdot r_1$. Since we also know that $m_0 = m_{SWORD} - m_{ROD} - m_1$, we have $m_1 = m_{SWORD} - \frac{m_{ROD}}{2} - \frac{m_{SWORD} \cdot r_1}{L}$.

In all this analysis, I've neglected to discuss the moment of inertia of the different masses around their own centers of mass. I've been treating them as point masses, which are infinitely small. However, each real weight you make, whether from lead, copper, or steel, will have some small moment of inertia of its own, before you even add it to the sword simulator. This effect is slight, and if you want to eliminate it just subtract the sum of the weights moment of inertias from the final moment of inertia you want to reach. There are textbook formulas for calculating the moment of inertia of different shapes, whether your weights are shaped like discs, spheres, or cylinders. For example, if your masses have a total moment of inertia that's 0.002 kg-m2 and your sword is supposed to measure 0.128 kg-m2, just make your placement calculations based on reaching 0.126 kg-m2.

Building Practical Simulators

I've illustrated one type of simulator, made from aluminum tubing. It has the advantage that steel rod can be cut to length and slide inside, with bolts or rivets pinning the rod in place. It might be easier to make the simulator from aluminum flats, just like the kind used in ARMA sparring swords. That type would use steel flats as weights, which could be bolted onto the aluminum flat. Another type I've made uses ¾" wooden dowels, with ¾" washers as the weights. The pommel washers are held on by a smaller washer, which is fastened to the end of the dowel with a wood screw. All other washers are held in place by wrapping duct tape around the dowel until it was thick enough to pin them in place. Regardless of how you make a simulator, make very sure that the tip weight can't fly off the end. If you are using aluminum tubing, crimp the tip so nothing can fly out.



The only other component to add to the simulator is the grip and cross guard. These can have their masses measured, and calculated into the final design. However, they form a supremely important part of the system, since it is through your hands that all these parameters are finally measured. Getting use to the feel of a great sword with a foil's grip really isn't helpful. Additionally, if you finger the cross guard, you may want to base your simulator on an aluminum flat, since in no way will a tube ever feel like a blade. Also, the closer the pommel approximates the shape of the real pommel, the better.

So we can calculate the required masses and positions to replicate the basic feel of any sword. This means we can go to a museum and measure the feel of an authentic sword, then convert this into a simulator design that anyone at home can easily assemble. This may help ease the difficulties the majority of sword practitioners have in knowing how particular types of swords are supposed to handle. After all, we're just now getting rid of the 20-pound swords, so we might as well be able to illustrate how the different types really handle, so we can avoid getting stuck with really horrible replicas. Another use for a simulator is to provide a hands-on illustration of how swords with exactly the same weights and balance points can feel drastically different. We encounter this effect all the time, but usually it's attributed to the skill of the smith, and surrounded in all kinds of mysterious terms. When you can show the same effect with a wooden dowel and some washers, the mystery fades. It's not the skill of the smith, it's the basic design that provides the answers to this mystery, which boils down to the mass distribution of the sword. I supplied four simulators to ARMA, each with exactly the same weight and balance point, yet a nearly four-fold difference in moment of inertia. The one with a tiny moment of inertia handles like an odd type of foil. If feels like a foil with a chicken shoved up to the guard. It feels heavy, yet is extremely maneuverable. The one with the highest moment of inertia feels almost like swinging a barbell, and can hardly maneuver at all. The other two are in between in their performance.

Chapter 2

Simple Motions

Acceleration of the Center of Mass

When a single force is applied to an object, the center of mass of the object accelerates according to Newton's law **F=ma**, even if the force is applied far from the object's center of mass. A very popular misconception arises when the force is applied perpendicularly to the end of a club, staff, or sword. All too often one hears that this force causes the object to rotate around its center of mass, and in a sense it is true. It is common in physics to break the resulting motion down into the linear motion of the center of mass, and then the separate rotation around the center of mass. However, the object is not using the center of mass as a pivot point, relative to its previous position. An object will always rotate around its center of mass in the absence of any applied force, but not really in the presence of an applied force. I think this has confused many people into thinking an object *always* pivots around the center of mass, which just isn't true. You hear versions of this misconception all over the place, even from influential professors.

Another related misconception is that if a force is applied far from the center of mass, such as perpendicularly to the end of a stick, the center of mass will accelerate less than if the force was applied directly at the center of mass. If you press the issue, most people will concede that the center of mass will move somewhat, but much less than it would have, since some of the applied energy goes into rotation, leaving much less available energy to go into the linear motion of the center of mass. What is actually happening is that the object rotates away from the impact so easily that very little force actually gets applied, and less force means less acceleration of the center of mass. It's a similar problem to applying force to a light object. For a given force, if the mass is small the acceleration has to be very large, and applying large accelerations by hand is difficult. This is probably what leads people to conclude that the center of mass accelerates less when force is applied to the end of a stick rather than to the stick's center of mass. All this becomes important when we start looking at how an object rotates during an impact, and it also comes into play when we look at how we apply forces to maneuver a weapon. So hang on through the math, though most of these equations will never be looked at again. They're just there to confirm that we're on the right track.

First, let's establish intuitively that applying a force perpendicular to the end of an object accelerates the object's center of mass. To show this is to lay a pencil on your desk, and then try to spin it with a strong flick at one end. The pencil of course careens off the desk, which it couldn't do if the center of mass didn't accelerate. If the impact only caused rotation, the pencil would have ended up spinning rapidly on top of the desk, not flung across the room. After all, the pencil can't fly into the floor and still leave its center of mass on the desk. So the center of mass does experience acceleration, which also means that the object didn't pivot around the center of mass during the application of the force. If it pivoted around the center of mass, which was initially stationary, then the center of mass would have remained stationary during the application of force, and certainly would have remained stationary after the impact was over, since the center of mass of an object doesn't undergo acceleration in the absense of an applied force. So the pencil's center of mass, and thus pencil, would have remained on the desk, spinning around like a propeller. Note, however, that once the force is removed, and the pencil is in uniform motion toward the floor, it does rotate purely around its center of mass, all the way to the floor. But it doesn't do this during the application of the force.

Now let's establish that the center of mass experiences an acceleration exactly as dictated by Newton's law. The contrary thought is that this acceleration must be somewhat less, because some energy goes into rotation. This failure of "common sense" arises from a confusion of the applied force with the applied energy, which definitely aren't the same things. First of all note that the chances of applying a force exactly in line with an objects center of mass are essentially zero (given enough decimal places), so there is always some rotation imparted by a force, however slight. If the object's rotation acted to decrease the acceleration of the center of mass, then in all real world cases the applied force would be greater than the mass times acceleration, giving us the formula $F > m \cdot a$, which is not quite kosher in Newtonian physics.

The confusion over energy goes away when we take a closer look at what's happening. Without the rotation the force is applied over some distance, normally given by the formula $s = \frac{1}{2} \cdot a \cdot t^2$.

The work done to the object, which is the energy input, is force times distance. But when the force is applied to some point that's not in line with the object's center of mass, the object rotates away from the applied force. Due to this rotation the force must be applied over a longer distance. This extra distance multiplied by the applied force is the rotational energy imparted to the object. Again, the same force is applied for the same duration, but more energy is imparted due to the longer distance of application.

Getting into the rotational mechanics, the applied force times the perpendicular distance **Q** to the center of mass gives the applied moment, **M=FQ**. The angular acceleration α is given by the formula $\Gamma = I_{COM}\,\alpha$, which is the rotational version of **F=ma**. We can measure the objects moment of inertia **I**, so we can calculate

$\alpha = \Gamma / I_{COM}$	angular acceleration	$\alpha = FQ/I_{COM}$
$\varpi = \alpha t$	angular velocity	$\varpi = FQt/I_{COM}$
$\theta = \frac{1}{2}\alpha t^2$	angular position	$\theta = \frac{1}{2} FQt^2 / I_{COM}$

For very small angles where $\sin(\theta)$ is approximately equal to θ the extra distance due to the rotation is $Q\theta$. Calculating this linear distance times force gives work input into rotation about the center of mass.

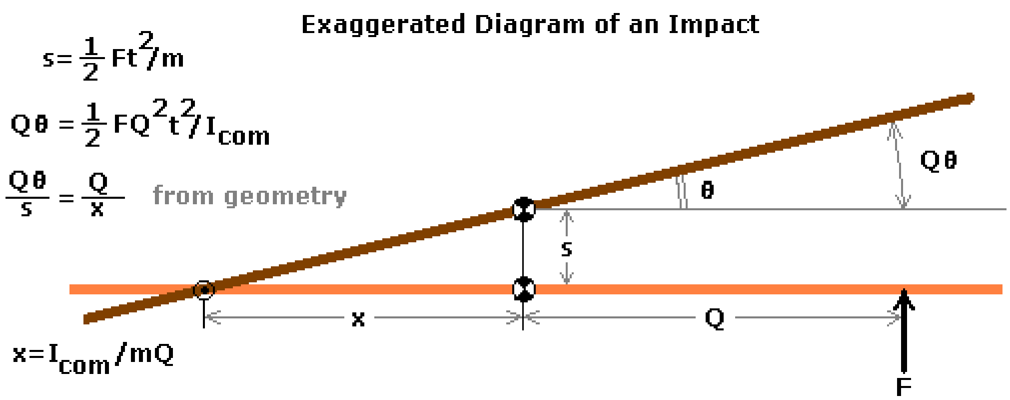
$W = FQ\theta$	work applied	$W = \frac{1}{2} F^2Q^2t^2/I_{COM}$

The rotational kinetic energy about the center of mass is given by

$KE = \frac{1}{2} I_{COM}\,\varpi^2$	kinetic energy	$KE = \frac{1}{2} F^2Q^2t^2/I_{COM}$

This is exactly what was calculated for the extra work done due to the rotation of the object. We just found where that extra energy comes from, and confirmed that **F=ma** for the object's center of mass.

Don't let the math scare you, since you'll probably never need to see these equations once we figure out what point serves as the axis of rotation during an impact.



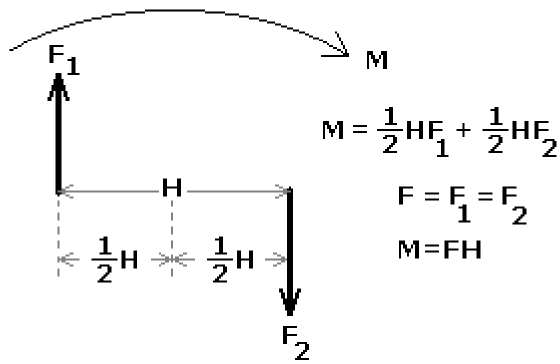
Here's an exaggerated diagram of an impact where θ is made large for clarity. The light brown stick is the initial position, the dark brown stick the position at the end of the impact event. The impact consists of the force F applied at distance Q from the center of mass (the little round symbol in the center of the stick). The center of mass accelerates away and travels the small distance s, which is equal to $\frac{1}{2} \cdot a \cdot t^2$, but since $a = \frac{F}{m}$ we also have $s = \frac{1}{2} \cdot \frac{F \cdot t^2}{m}$. From the preceding section's equations we have $\theta = \frac{F \cdot Q \cdot t^2}{I_{COM}}$, so $Q \cdot \theta = \frac{F \cdot Q^2 \cdot t^2}{I_{COM}}$. From basic geometry (hope you didn't sleep through class) we can see that the distance $Q \cdot \theta$ is to distance Q as distance s is to x. Solving this ratio algebraically we get $\theta = \frac{s}{x}$, and finally an equation that relates mass, inertia, impact point, and axis of rotation. Rewritten four different ways, since it's so handy, we have our useful result.

$$x = I_{COM}/mQ$$
$$Q = I_{COM}/mx$$
$$Qx = I_{COM}/m$$
$$I_{COM} = Qmx$$

But just make sure you remember the main point. Objects do NOT rotate around their center of mass when a single force is applied. They rotate around their center of mass in the absence of applied force, but not during the application of the force.

Applied Couples

Let's see what happens when you apply two forces to the weapon, which is what your forearm does to the handle. Starting with the simple case of two equal and opposite forces, separated by some distance, such as your forefinger to your pinky. These forces form what's called a couple. The couple applies a moment of force to the weapon, and that moment tries to rotate the weapon about its center of mass. Note that a single force can never rotate the weapon about its center of mass, but a couple can do nothing but rotate it about its center of mass. This distinction is important and should be remembered.



Also note that the forces making up the couple are equal and opposite. They cancel each other out, with regards to accelerating the object in any particular direction. Since they present no unbalanced force to the object, the object's center of gravity can't be accelerated, since with no applied force, the equation $F = m \cdot a$ gives no acceleration. Since the center of mass isn't accelerating, it isn't going to move anywhere. So any rotation that occurs must be around the stationary center of mass. This may seem a bit odd, since when you apply a torque to your handle the weapon rotates around your hand, not the center of mass. But what's happening is that you're applying an unbalanced force, along with the couple. In the case of a two handed grip it means one of your hands is applying more force than the other hand. If the blade is horizontal and you rotate it upward to vertical, you must be applying an overall upward force to raise the height of the center of mass. This is of course neglecting gravity, which for simplicity is being ignored in these calculations so that they apply to horizontal, vertical, and any other situations. Once you're looking at a specific cut toss in gravity acting vertically downward on the center of mass of the weapon.

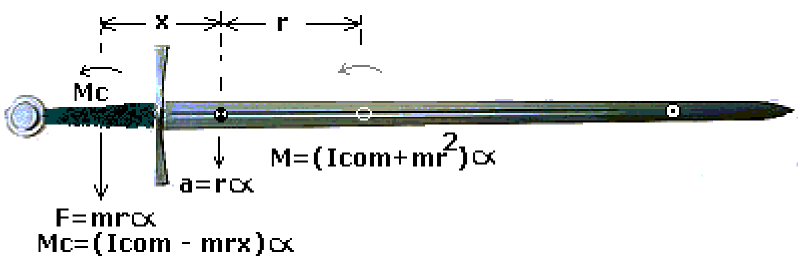
Try this experiment. Hold your weapon vertically by placing your hand at the center of mass. Then rapidly rotate it clockwise and counter clockwise by some small angle, all while feeling just how much effort your forearms are putting into it. Now move your hand down to the handle, and consciously rotate the weapon about its center of mass. You now have to move your forearm back and forth to track the motion of the handle, but the forearms are doing exactly the same work as before. You may have to lay a finger of your other hand at the center of mass to keep yourself steady when you try this second motion. What's odd is that where you apply the couple along the weapon doesn't matter, as the resulting motion is the same. For analysis you can slide the couple to any point along the weapon to make the math easier.

Try another experiment. Compare the effort required in rotating the weapon about its center of mass with the effort required in rotating it about the handle. Rotating about the handle is much more difficult for two reasons. One is that the moment of inertia at the handle is higher than the moment of inertia about the center of mass. This extra inertia requires more torque to yield the same accelerations. Your forearm has to work harder to overcome the extra moment of inertia. The other thing that hampers the motion is that now you have to accelerate the center of mass of the weapon. When you were rotating about the center of mass this extra force wasn't required.

Please note that the torque used to accelerate the center of mass at some given rate depends only on weight and balance point, not on the moment of inertia of the weapon. The moment of inertia determines the magnitude of the couple used to rotate the weapon about the center of mass. Combined, these parameters determine how easy it is to move the weapon, and also how the weapon reacts to the applied forces.

Simple Maneuver Equation

These equations boil down to the following diagram, which again neglects gravity for universal application. It also won't take things through a real swing, as centripetal forces, etc. are not included. This just covers the simple basics of moving a weapon (in this case a sword) from a standing start. It also doesn't cover the thrust components, just simple rotations about a point in line with the blade.



Unbalanced force F

This picture describes the applied forces required to apply an angular acceleration α to a weapon about some arbitrary point located at distance r forward of the center of mass (r can also be negative). Geometry dictates that the center of mass must accelerate according to its distance from r, so its linear acceleration is given by $a_{COM} = r \cdot \alpha$. From Newton we know that the unbalanced applied force necessary to cause this acceleration is given by $F = m \cdot a_{COM}$, where as just stated $a_{COM} = r \cdot \alpha$, so $F = m \cdot r \cdot \alpha$. Our earlier analysis of the acceleration of the center of mass showed that it doesn't matter where this force is applied to the weapon, the center of mass will always accelerate the same way, which is why I previously went through all the math on it. Since you supply this force, we will move the force to the hilt. Exactly where to move it to on the hilt is a bit more complicated, as the force might come from your first knuckle or your pinky knuckle, or thereabouts, assuming we're doing a single handed analysis. Once you decide where to apply this force, let x be its distance from the center of mass.

Moments

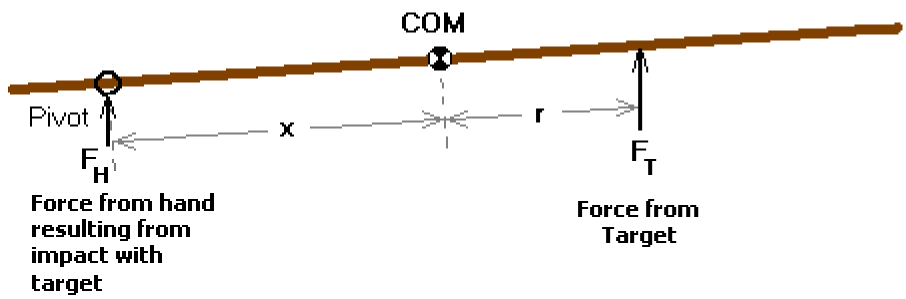
Whichever point you choose for x, keep in mind that this force now also applies a torque to the weapon, just as applying a force to a wrench does. Since the force is applied at distance $x + r$ from the point of rotation, we have a simple torque from the hand given by $\Gamma_O = F \cdot (x + r)$. But since $F = m \cdot r \cdot \alpha$ we end up with this moment or force as $\Gamma_O = (m \cdot r \cdot x + m \cdot r^2) \cdot \alpha$. Given that the weapon's rotational acceleration α about the point at distance r from the center of mass must result from a torque $\Gamma = I_r \cdot \alpha$, and $I_r = I_{COM} + m \cdot r^2$, we have the total torque Γ that's applied. From this moment we can subtract the moment resulting from the unbalanced force F applied to the handle to move the center of mass, which is just a single part of the moment of force necessary to rotate the weapon. This yields the couple that must be applied to the hilt, which would be $\Gamma_C = \Gamma - \Gamma_O$, which is the moment necessary for the result minus the moment applied to accelerate the center of mass. Using our previously calculated formulas, the moment applied to the hilt is $\Gamma_C = (I_{COM} + m \cdot r^2) \cdot \alpha \quad - \quad (m \cdot r \cdot x + m \cdot r^2) \cdot \alpha$, obviously the term $m \cdot r^2 \cdot \alpha$ cancels out, so that leaves $\Gamma_C = (I_{COM} - m \cdot r \cdot x) \cdot \alpha$. For single-handed weapons the forearms must apply this moment. So now we essentially have the formulas for what your forearms do (Γ_C) and what the rest of your body does (F).

$$\mathbf{F = m\, r}\, \alpha$$
$$\Gamma_{\mathbf{c}} \mathbf{= (I_{COM} - mrx)}\, \alpha$$

Chapter 3
Simple Impacts

Reaction Forces at a Fixed Pivot Point

Suppose we fix the axis of rotation, so that it doesn't move, as if it were a bearing. This might serve as a greatly simplified version of your hand during a strike. What forces are imparted to an immovable hand during an impact at some distance from the center of mass? Here is a diagram of such an impact.



Unbalanced Forces $\mathbf{F_T}$ and $\mathbf{F_H}$

Here an impact with the target applies a force F_T to the staff, but this also produces a reaction force F_H from the resulting pivot point, which is your hand. The center of mass is accelerated by the sum of both of these forces, so $F_T + F_H = m_{SWORD} \cdot a_{COM}$, and $a_{COM} = \frac{F_T + F_H}{m_{SWORD}}$. But the center of mass moves around the pivot point, and by simple geometry we know that $a_{COM} = x \cdot \alpha$. So we have an angular acceleration α, where $a_{COM} = x \cdot \alpha$, with $F_T + F_H = m_{SWORD} \cdot x \cdot \alpha$, and thus $\alpha = \frac{F_T + F_H}{m_{SWORD} \cdot x}$. This may seem a bit confusing at first, since the angular acceleration is not only dependent on the force applied at the impact point, but also on the force applied to the fixed pivot. But if you had a stick mounted in a bearing, pushing on the bearing won't rotate the stick. This confusion comes from thinking of applying force to the bearing. Since the bearing is fixed in place, it will push back against your hand, with a force equal and opposite any forces that you apply to it. In short, your pushing against the bearing's mounting, and the mounting is pushing back. This doesn't tell us anything about the forces that the bearing is applying to the stick. You know that if you push against the stick, very close to the bearing, then the stick is applying a force to the inside of the bearing. This internal force must be accounted for, and we've done it with the equation shown above.

Moment $\Gamma_{\mathbf{P}}$ about the Pivot

The reaction force at the pivot point can't make any torque contribution to rotation about the pivot point, since the distance from the pivot point of the force's application is zero. The torque component of the rotation must come only from the force applied by F_T, so the torque applied to the hand is $\Gamma_H = F_T \cdot (x + r)$. But the torque is also equal to the moment of inertia about the axis of rotation multiplied by the angular acceleration, so $\Gamma_H = I_X \cdot \alpha$, where I_X is the sword's moment of inertia about the pivot point, or hand. This is calculated using the parallel axis theorem, as always, and gives us $\Gamma_H = \left(I_{COM} + m_{SWORD} \cdot x^2\right) \cdot \alpha$. Substituting in the equation for the torque about the pivot point, we get the equation $F_T \cdot (x + r) = \left(I_{COM} + m_{SWORD} \cdot x^2\right) \cdot \alpha$.

Combining the Equations

Substituting in the value of α, from the unbalanced force calculations, gives us the following.

$$F_T \cdot (x + r) = \frac{\left(I_{COM} + m_{SWORD} \cdot x^2\right) \cdot (F_T + F_H)}{m_{SWORD} \cdot x}$$

$$F_T \cdot x + F_T \cdot r = \frac{(F_T \cdot I_{COM} + F_H \cdot I_{COM}) + (F_T + F_H) \cdot m_{SWORD} \cdot x^2}{m_{SWORD} \cdot x}$$

$$F_T \cdot x + F_T \cdot r = \frac{(F_T \cdot I_{COM} + F_H \cdot I_{COM})}{m_{SWORD} \cdot x} + \frac{(F_T + F_H) \cdot m_{SWORD} \cdot x^2}{m_{SWORD} \cdot x}$$

$$F_T \cdot x + F_T \cdot r = \frac{(F_T \cdot I_{COM} + F_H \cdot I_{COM})}{m_{SWORD} \cdot x} + (F_T + F_H) \cdot x$$

$$F_T \cdot x + F_T \cdot r = \frac{(F_T \cdot I_{COM} + F_H \cdot I_{COM})}{m_{SWORD} \cdot x} + F_T \cdot x + F_H \cdot x$$

$$F_T \cdot r = \frac{(F_T \cdot I_{COM} + F_H \cdot I_{COM})}{m_{SWORD} \cdot x} + F_H \cdot x$$

$$F_T \cdot \left(r - \frac{I_{COM}}{m_{SWORD} \cdot x}\right) = F_H \cdot \left(\frac{I_{COM}}{m_{SWORD} \cdot x} + x\right)$$

Multiplying both sides by $m_{SWORD} \cdot x$, we get another form of this equation.

$$F_T \cdot (m_{SWORD} \cdot x \cdot r - I_{COM}) = F_H \cdot \left(m_{SWORD} \cdot x^2 + I_{COM}\right)$$

Some people take the direction of the force F_H to be opposite that of the impact force. This simply results in another form of the equation.

$$F_T \cdot (I_{COM} - m_{SWORD} \cdot x \cdot r) = F_H \cdot \left(I_{COM} + m_{SWORD} \cdot x^2\right)$$

$$\frac{F_H}{F_T} = \frac{(I_{COM} - m_{SWORD} \cdot x \cdot r)}{\left(I_{COM} + m_{SWORD} \cdot x^2\right)}$$

Setting F_H to Zero

We want to know where the least hand shock occurs, so setting F_H to zero must mean that the numerator in the previous equation is also zero. This means that for minimal hand shock $r = \frac{I_{COM}}{m_{SWORD} \cdot x}$. So we find that for minimal hand shock, the point of impact should be located at a distance from the center of mass, determined by the formula $r = \frac{I_{COM}}{m_{SWORD} \cdot x}$. This point seems to crop up everywhere, so it must be of some importance. In dynamics this point is called the percussion point, relative to point *x*. A force applied at this point will cause a rotation about the point located distance *x* from the center of mass, with no torque applied to point Yes, we've mathematically found that point you're so familiar with by hitting the side of your blade. Unfortunately that point your looking at by banging on your blade is not the percussion point. Big oops all around. In case you're wondering if this percussion point, also known as the point of percussion or center of percussion, has anything to do with hand shock, keep in mind that physicists also call it the vibration point.

The Definition of Percussion

You might wonder how swordsmen, baseball players, and physicists came to use the same word for the same thing. The reason is that they got it from the same course of analysis, back in the 1500 and 1600's. Back in those days percussion was a hot topic of discussion amongst theoretical physicists like Galileo Galilei, Giovanni Borelli, Christopher Wren, Christiaan Huygens, John Wallis, and Isaac Newton. Especially important were the years 1667, when adsf took another look at Galileo's work, 1668 when Christopher Huygens made some theoretical advances with studies on pendulums, and 1669 when the eminent mathematician John Wallis presented some of his results.

Percussion: 1. The striking of one body with or against another with some degree of force, so as to give a shock; impact; a stroke, blow, knock. Usually in reference to solid bodies. Chiefly in scientific use.

Also in use at the time was a definition that meant to make a sound in the air by making a blow, or a sudden movement of air. So its usage was probably similar to the modern words bang, crunch, biff, smack, or bam, and though we have the phrase crunch time, we don't have any phrase like biff point. Another set of meaning comes from the study of Chiromancy, or palm reading, in which the lower side of a closed fist was known as the pomell or percussion. Other meanings for percussion came later, such as the musical meaning of percussion instruments starting in the 1770's, a firearms percussion cap starting in the early 1800's, and the medical meaning of tapping and listening beginning in the 1840's.

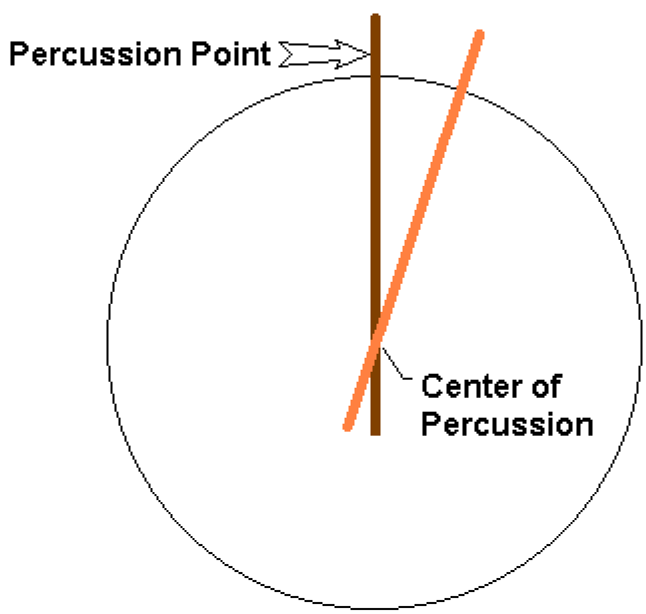
There is a related verb, ***percuss***, which means to strike or thrust through, from Latin, which the Oxford English Dictonary's timeline lists from 1560 to 1694. It also gives us the tenses ***percussed*** and ***percussing***. We had the adjective ***percutient***, also often spelled ***percussent***, which refers to the striking body or agent, so we can talk about the importance of the length of the *percutient* body relative to the mass of the target.

To make a resultant return blow, or to have your own blow make an elastic collision with a resilient body and then recoil back at you, was known as a ***repercussion***. When you're test cutting, you always need to beware of the possible repercussions. Another fun word we've lost is ***resilition***, which is the property that explains why certain objects are resilient. Resilition is a very good property for a sword to possess.

Here is a smattering of citations from the Oxford English Dictionary, just for completeness.

1544. Phaer *Regim Lyfe* C vij. "Sometyme the sayde payne of the eye cometh by percussion or strykyng."
1626. Bacon *Sylva* 190 "Where the Aire is the Percutient, … against a hard body it never giveth an exterior sound."
1656. tr. *Hobbe's Elem. Philos.* 347 "The velocity of the percutient is to be compared with the magnitude of the ponderant.
1666. *Phil. Trans.* I 306 "The Vehemence of the Percussion depends as much upon the length of the percutient Body, as upon the velocity of Motion"
1669. *Phil. Trans.* IV 1088 "The Doctrine of Percussion on which depends that of the Cuneus or Wedge." Which was a review of John Wallis' book "Mechanica, sive de motu" which is considered the most thorough study of mechanics and motion prior to Isaac Newton's "*Principia"*.

So if percussion just means impact, then *percussion point* literally just means *impact point*. As a result of the impact, the *percutient* body rotates in a small arc, which can be geometrically illustrated. Don't bother about the circle not coinciding with the percussion point. I could draw any number of circles, all with the same center, and each illustrating the motion of some random point on the sword edge.



This resultant rotation, being circular, has a well-defined center, which is the center of the percussion, or more formally, the *center of percussion*. But don't think that the object continues to pivot around this point *after* the force is removed. In the absense of an applied force, all objects rotate around their center of mass. In short, the percussion point and center of percussion only have physical meaning during the impact.

More on Pivot Forces

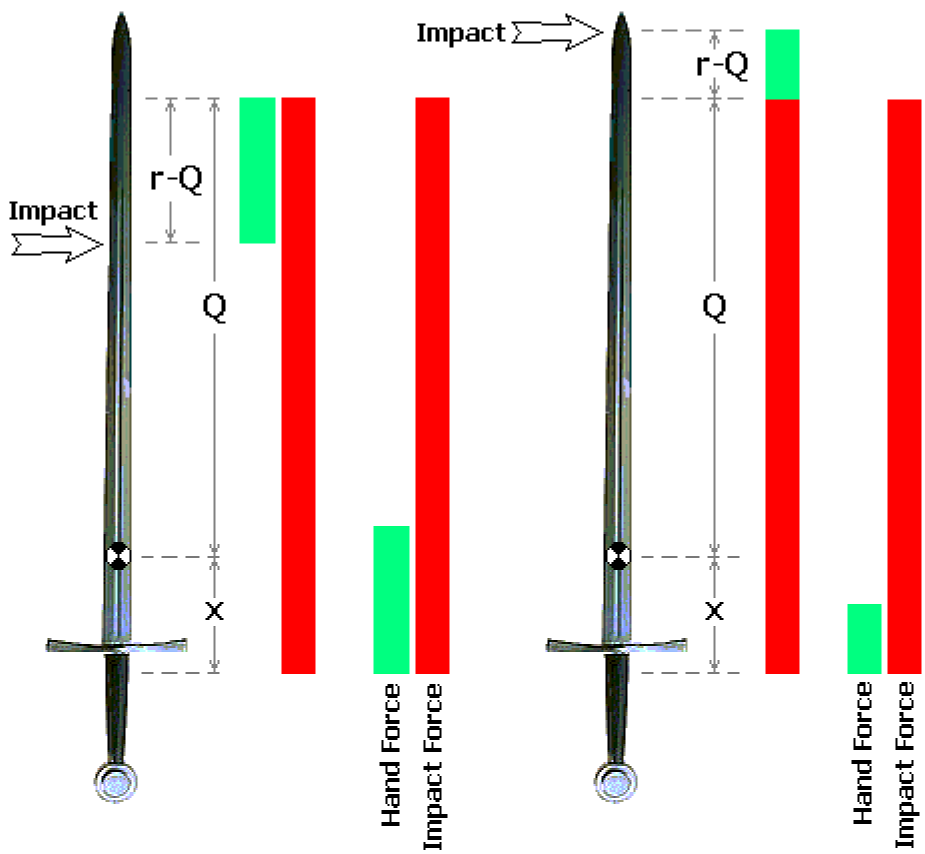
If we recognize that the equation for the percussion point, $Q = \frac{I_{COM}}{m_{SWORD} \cdot x}$, also appears in the equation for the hand shock versus impact force, we can simply the equation further.

$$F_T \cdot \left(r - \frac{I_{COM}}{m_{SWORD} \cdot x} \right) = F_H \cdot \left(\frac{I_{COM}}{m_{SWORD} \cdot x} + x \right)$$

$$F_T \cdot (r - Q) = F_H \cdot (Q + x)$$

$$\frac{F_H}{F_T} = \frac{(r - Q)}{(Q + x)}$$

Graphically, we know that the distance $Q + x$ is simply the distance from your hand to the percussion point relative to your hand. Since we can easily find where this is, on any given sword, we can picture it quite nicely. Also, since we know where the percussion point is located, and we know where the impact point is, we can see the distance from the percussion point to the impact location, or $r - Q$. To illustrate this, look at the following drawing.



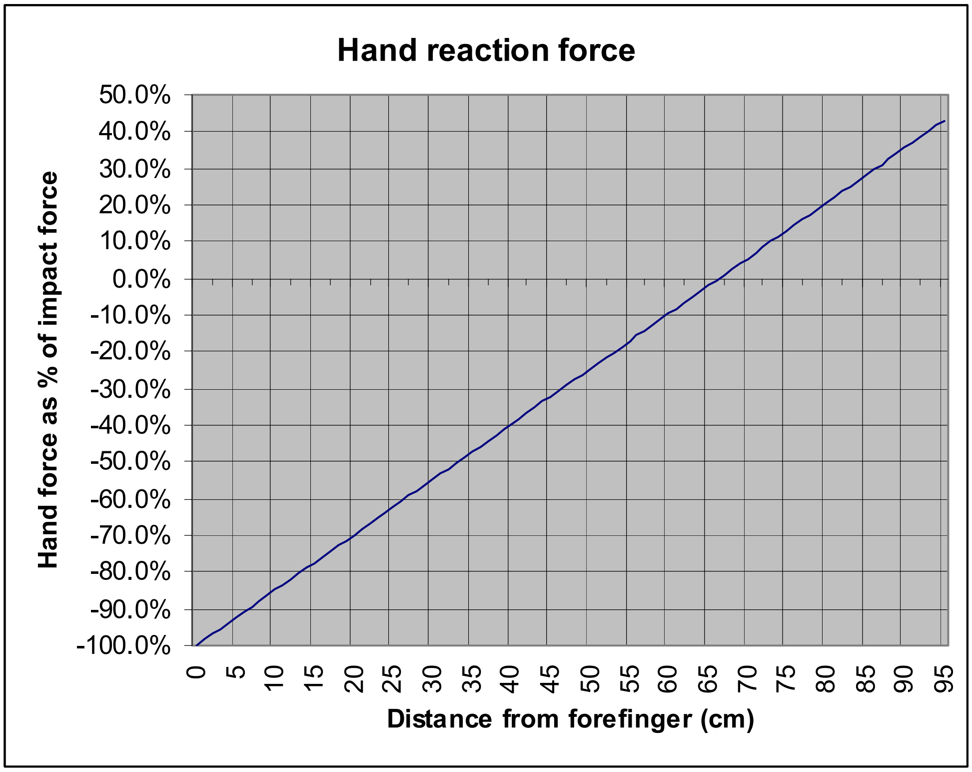
A bit of percussion point algebra

$$Q = I_{COM}/mx$$
$$Q + x = I_{COM}/mx + x$$
$$Q + x = I_{COM}/mx + x\ (mx/mx)$$
$$Q + x = I_{COM}/mx + mx^2/mx$$
$$Q + x = (I_{COM} + mx^2)/mx$$
$$Q_P = I_P/mx$$

So the hand shock builds up linearly with distance from the percussion point, and taking **r-Q** to be the distance you've missed the sweet spot by, the hand forces are to the impact forces as this miss distance is to the distance from your hand to the sweet spot. So these forces stay in geometric proportion to the weapon. Weapons with a longer effective length (distance from hand to percussion point) are less sensitive to missing the sweet spot than are shorter weapons. The miss distance is considered only in proportion to the hand to percussion point distance.

Hand Reaction Forces

I have an Austrian heavy cavalry saber from the 1850's. Since I've measured all its relevant parameters (Now you know how I've been spending my free time) Here's a graph of this hand force/impact force versus impact point.



This was calculated based on the percussion point relative to the forefinger. Note that the negative values for hand force means the grip kicks back into your palm, whereas the positive values have the grip kicking into your fingers. This gives one of the ways of finding the percussion point. Strike an object. If the handle first kicks into your fingers, the impact was out past the percussion point. If the handle kicks back into your palm, then the impact was inside the percussion point. Richard Burton almost explains this procedure in this book "The Book of the Sword". He merely says you cut inch-by-inch looking for the point where the impact doesn't jar your hand. He should've gone a bit further and explained how you know which direction to move the impact, if he even knew. However, Sir Richard talked about swords with just about everyone, and if you could really find the percussion point by slapping the side of the blade, I think someone would've told him that. He completely omits the slap test, so it was likely invented sometime later.

Also note that the percussion point of this saber is about two thirds out from the forefinger to the tip. This is the striking point that everyone trained in cavalry saber was taught, and it's pretty much the place where we are all told the sweet spot lies. It also pretty much coincides with the node of oscillation you get when you slap your blade. The prevalence of cavalry training in recent swordsmanship may lie at the heart of the complete misunderstanding of the percussion point.

Another interesting thing about the graph is its linearity. The hand shock forces start out as a very large percentage of the target force, for strikes near the hilt, then linearly diminish to the zero at the percussion point, then linearly increase again. There is no magic as you cross the percussion point distance. If you want hand forces to always be less than 15% of impact forces then on this particular Austrian saber you have a zone of plus or minus 10 cm from the percussion point in which to strike. This gives a 20 cm (5-inch) zone of the blade that meets the given specification. If you stray out of this zone, nothing really happens other than a slightly higher hand shock.

Striking at the percussion point gives no hand shock, but missing with it is only as bad as the linear distance with which it was missed. This correlates will with Hand Reinhardt's descriptions of the percussion point. He says that there is no magic spot. If you miss it by a little the effect isn't large. Anywhere around it cuts very well. This also correlates with another of Hank's observations that to strike well, you swing hard, and when you hit your hand feels nothing. If you hit far back from the percussion point the handle will kick back into your palms, which may feel powerful to some, but may not actually be hitting as hard as with the percussion point. Similarly, as you accelerate the sword toward the target, you will feel the handle pushing back into your palms, due to Newton's third law. You're applying force to the handle so the handle applies force equal and opposite into your palm. If you reach peak rotational speed just prior to impact, which will make the strongest strike, this force will disappear immediately before impact. So just prior to and during impact your hand should feel nothing.

This aspect of hitting has misled many baseball players. In batting practice they will occasionally swing correctly and really smack the ball, but they think it feels weak because they didn't have that strong feeling of "power" in their hands during impact. That "power" is the bat firmly pushing into their palms, which indicates that at impact they were still accelerating the bat, and thus hadn't yet reached peak bat speed. Then they change their swing to get the powerful feeling back, which means they aren't getting the bat up to speed, and return to poorer hitting.

Chapter 4
Complex Motions

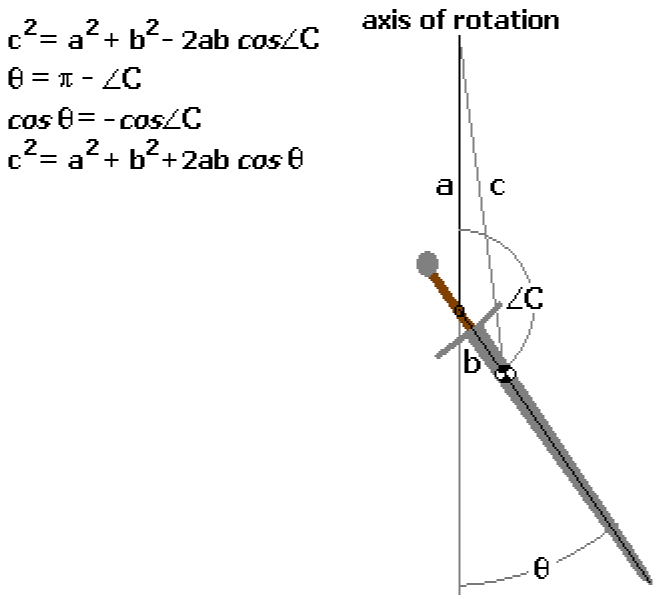
Centrifugal Forces During a Swing

One thing that swings have in common is that the weapon is placed under intense angular acceleration, and centrifugal acceleration, by the arc of the hand's travel. On a very simplistic level the hand's angular acceleration ω_H , is determined by the weights and inertias of the sword and sword arm. Also, but somewhat simplistically, the distance from the arms axis of rotation, be it the shoulder or elbow, to the sword's axis of rotation (the sword hand), which I'll call r , and the hand's angular velocity ω_H determine the centrifugal acceleration that determines the sword's period of oscillation. Let's focus on the way the weapon will act under the influence of a constant centrifugal acceleration.

The sword, if displaced from a direct line leading from the arms axis of rotation, will act like a pendulum under the influence of the centrifugal acceleration instead of gravity. The centrifugal acceleration, given by the formula $a_C = \omega_H^2 r$, does not act quite like gravity. One might think that the period could be found by determining the centrifugal acceleration felt by the swords center of mass, but this is not so. The formula works out to be the same as a normal pendulum, but with the acceleration due to gravity replaced by the centrifugal acceleration of the sword's pivot point, which is the hand.

Think about a pendulum pivoted on the axis of a rotating shaft. You can't displace it from straight out, as this would just move it forward or back along the same circular path, which it was already traveling along. So in this case, no matter what centrifugal forces are felt by the pendulum, the pendulum will not "swing". Likewise, if the pendulum is pivoted some very small distance from the axis of rotation, it will swing only very slowly. This is unlike the case of a pendulum under the action of gravity, which always pulls vertically downwards. When the pendulum is pivoted very close to the axis of rotation, the forces are aligned on that axis. So even for large displacements of the pendulum, for force remains close to the line from the pivot point to the pendulum. Only when the distance r gets large does the pendulum start to speed up. As the distance r becomes very large, the pendulum starts to behave as if the centrifugal forces were gravitational.

In a gravitational field the potential energy of a mass is given by $PE = \frac{1}{2}mgh$, where m is mass, g is the acceleration due to gravity, and h is height. For centrifugal accelerations the potential energy is given by the equation $PE = \frac{1}{2}m\omega^2 r^2$, where here r is the distance to the sword's center of mass. If the distance from the axis of rotation is given by to the pivot point is given by a , and the distance from the pivot point to the sword's center of mass is given by b , then the square of the distance from the axis of rotation to the center of mass is given by $r = \sqrt{a^2 + b^2 + 2ab\cos\theta}$, where θ is the angle from the center of mass, to the line between the axis of rotation and the pivot point.



So the potential energy of the sword is given by $PE = \frac{1}{2} m\omega_H^2 (a^2 + b^2 + 2ab\cos\theta)$. The kinetic energy is given by $KE = \frac{1}{2} I_H \omega_S^2$, where ω_S is the angular velocity of the sword about its pivot point, (independent of ω_H, the angular velocity of the hand) or just $d\theta/dt$. If the forearm isn't applying any torque, then due to the conservation of energy, the sum of the kinetic and potential energies must be a constant. Solving this equation, we get a formula for the period of the sword, as it swings like a pendulum. This equation comes out to be the same as the normal equation of a pendulum, where the acceleration due to gravity is replaced by the centrifugal acceleration at the sword's pivot point, your hand.

For small displacements the period τ of a simple pendulum is given by the equation $\tau = 2\pi\sqrt{\frac{Q_P}{g}}$, where Q_P is the distance from the hand to the hand's percussion point, and g is the acceleration due to gravity. Substituting in the equation for the centrifugal forces in the swing we get a period given by $\tau = 2\pi\sqrt{\frac{Q_P}{\omega_H^2 r}}$. This reduces to $\tau = \frac{2\pi}{\omega_H}\sqrt{\frac{Q_P}{r}}$. The time taken to swing from an initial displacement to the centered position is of course one fourth of the total period, so this time is given by $\tau_{1/4} = \frac{\pi}{2\omega_H}\sqrt{\frac{Q_P}{r}}$. The angle covered by the hand during this time is given by $\theta_{1/4} = \omega_H T$, but the term ω_H cancels, so $\theta_{1/4} = \frac{\pi}{2}\sqrt{\frac{Q_P}{r}}$. This is how much angle your hand covers while the sword rotates from its initial cocked position, however far back that is, to it's position aligned with the axis of rotation, which should be roughly pointing straight out from your shoulder.

Note that this angle, which your hand travels while the sword comes around, doesn't depend on the initial cocked angle of the sword, nor does it depend on the angular velocity of your hand. It depends only on the square root of the ratio of Q_P/r, where again r is the distance from the axis of rotation to your hand, which is basically your arm length. This makes a great deal of sense. We either like the way a sword swings or we don't. We don't encounter a sword that feels good in an easy swing, but changes the way it behaves for a hard swing. The fact that the angular velocity of the hand is out of the equation for how the sword behaves thus sounds correct. Another thing to note is that the radius of your swing is mostly determined by your arm length and how you bend your arm, plus what your body and shoulder are doing. The square root of a sword's hand to percussion point distance, $\mathbf{Q_P}$, determines the sword's natural rotation during a swing. The sword's moment of inertia about the grip, $\mathbf{I_X}$, determines how much the arms can adjust the natural rotation. The swords mass determines how fast the hands can accelerate the sword through its arc. Also of note, is that we don't use the center of mass location for anything but the basic math. The sword's center of mass location isn't felt during a swing.

There is another factor contributing to the swing behavior, the hand's radius of motion. Looking at the equation for the angle covered by the hand as the sword rotates into striking position, $\theta_{1/4} = \frac{\pi}{2}\sqrt{\frac{Q_P}{r}}$; we see that the square root of the hand radius is as important as the percussion point distance. Swinging through an arc of larger radius lets the sword naturally align itself, all during a smaller angle covered by the hand. However, the distance that the arm must travel is greater, as this distance varies linear with swing radius, but the formula indicates that the resultant decrease in required swing angle only decreases with the square root of the swing radius. So big, sweeping swings don't have to travel as great an angle, even though the hands must cover a greater travel distance around the arc. More fully, the required travel distance of the hand increases, with the square root of the radius of the hands travel about the shoulder, whereas the angle about the shoulder that the hand must travel, decreases with the square root of this radius.

Let's assume a range of hand speeds, from 15 to 30 miles per hour, and calculate some of these numbers on a range of hypothetical swords. I'll give an arm length of 0.60 meters, assume no shoulder motions, and assume the arm is fully extended the whole swing. The swing is also considered to be at a constant rotational speed. This is of course a very artificial set up, but it makes the math easy. If pursuing this deeper is warranted then we'll have to do some serious modeling of swings using high speed video systems to record some raw data. But here's the data for this simplistic case.

Hand Velocity (mph)	Hand Velocity (m/sec)	Angular Velocity of Hand (rad/sec) about shoulder r = 0.6 meters	Centrifugal Acceleration at hand (m/sec^2)	Centrifugal Acceleration at hand G's	Full Period $\mathbf{Q_P}$=0.6m (seconds)	1/4 Period $\mathbf{Q_P}$=0.6m (seconds)	Hand angle travelled for 1/4 period (degrees)
15	6.71	8.94	48.0	4.9	0.703	0.176	90.0
20	8.94	11.92	85.3	8.7	0.527	0.132	90.0
25	11.18	14.90	133.2	13.6	0.422	0.105	90.0
30	13.41	17.88	191.9	19.6	0.351	0.088	90.0

This shows that indeed, according to the simplistic formulas, the angle traveled by your hand before the sword moves from it's cocked position to its aligned position is constant, regardless of the hand's swing velocity. The incredibly large values for the centrifugal accelerations felt by your hand are also quite amazing. When you think about crushing G-forces that fighter pilots deal with, look at the vastly larger ones that we commonly inflict on our limbs!

The percussion point distance used for this calculation (about 24 inches) is pretty typical of a cavalry saber, and probably not too far off of a single-handed medieval short sword. Let's plug in some other percussion point lengths and see how the data changes.

Velocity (mph)	Hand Velocity (m/sec)	Angular Velocity of Hand (rad/sec) about shoulder r = 0.6 meters	Centrifugal Acceleration at hand (m/sec^2)	Centrifugal Acceleration at hand G's	Full Period Q=0.9m (seconds)	1/4 Period Q=0.9m (seconds)	Hand angle travelled for 1/4 period (degrees)
30	13.41	17.88	191.9	19.6	0.430	0.108	110.2

Above we have a sword with a 50% larger hand to percussion point distance. It's probably somewhat close to the value for a medieval long sword. It takes a bit longer to naturally come around, about 20 degrees, but you've now got two hands on the hilt that can apply an immense torque to make up for the slower natural rotation.

Velocity (mph)	Hand Velocity (m/sec)	Angular Velocity of Hand (rad/sec) about shoulder r = 0.6 meters	Centrifugal Acceleration at hand (m/sec^2)	Centrifugal Acceleration at hand G's	Full Period Q=1.5m (seconds)	1/4 Period Q=1.5m (seconds)	Hand angle travelled for 1/4 period (degrees)
30	13.41	17.88	191.9	19.6	0.556	0.139	142.3

Here is an immense hand to percussion point distance of about 5 feet! As this implies a total sword length of over 6 feet, possibly more, it most likely represents a screwed up sword design. Yet it only takes about 32 degrees longer to come around naturally. You would probably notice this slowness, and John Clements surely would, but many who hadn't handled real medieval swords might take this as a normal behavior. Especially when you have a big two handed grip on this mighty sword.

As we make x very small, actually moving our hand near the center of mass, Q_P of course must become very large. When that happens the sword loses any natural inclination to rotate on around. This is shown in the following example.

Velocity (mph)	Hand Velocity (m/sec)	Angular Velocity of Hand (rad/sec) about shoulder r = 0.6 meters	Centrifugal Acceleration at hand (m/sec^2)	Centrifugal Acceleration at hand G's	Full Period Q_P=10m (seconds)	1/4 Period Q_P=10m (seconds)	Hand angle travelled for 1/4 period (degrees)
30	13.41	17.88	191.9	19.6	1.434	0.359	367.4

Here you can easily see that the sword will need to be swung a full circle before the strike can be delivered. The only way to handle this sword would be massive application of torque from your arms. This sword just wouldn't feel right to anyone. But it does show that this natural swing isn't very sensitive to percussion point distance, at least not as sensitive as you'd find from striking targets. Sword's with percussion points quite a bit past the tip will still swing, if not quite as nimbly. This is because this swing angle is only affected by the square root of the percussion point distance, whereas the striking point behavior is affected directly by this distance. The striking behavior is more sensitive to percussion point changes than swing behavior is.

Also of note is what happens when Q_P stays the same and x gets large, as with a hand axe or mace. This happens because these weapons have their center of mass is out near the far end. In this case even if Q_P is equal to some given sword the larger value for x will make the weapon naturally come around faster than the sword. On the down side the axe or mace will have a very high moment of inertia, so your forearms will be taxed in maneuvering.

Another interesting aspect of these equations comes from the potential energy of the sword if held cocked during the swing.

Conservation of Angular Momentum

The formula $H_O = I_O\omega$ gives the angular momentum H of the sword about some axis O, where I_O is the sword's moment of inertia calculated about the axis O. As the equation shows, angular momentum is moment of inertia about the axis multiplied by the sword's angular velocity ω about the axis. An impact, when the momentum of both the target and the swordsman are taken into account, conserves both the angular and linear momentum. The angular momentum of the sword is changed by the angular impulse imparted by your arm. The angular impulse delivered by the arm is calculated as torque multiplied by time, which also works out to exactly be the change in angular momentum of the sword.

Suppose your swing has reached its peak angular velocity ω, at which point you're neither accelerating, nor decelerating the sword. The means your arm is not applying any angular impulse to the blade, and thus the angular velocity of the blade will be conserved. If at this point your natural biomechanics pulls the sword back toward you, decreasing the swing radius, then by the parallel axis theorem the moment of inertia must decrease. Since in this case, the angular momentum $H_O = I_O\omega$ must be constant, and I_O has decreased, then the angular velocity ω must increase. Since the increase in ω is linear with the decrease in I_O, and the kinetic energy is given by $KE = \frac{1}{2} I_O \omega^2$, the sword's kinetic energy must be increased. The extra energy was supplied to the sword by the work done to pull it inwards, against the intense centrifugal forces of the swing. If part of this work is supplied by the your bones tightening the arc of the swing, or by large trunk muscles that weren't otherwise being used in the swing, then essentially, this extra energy is a freebie.

However, I would like to caution those who might use this bit of knowledge and screw up their swings. Just as the best way to beat an opponent at golf is to give him a stack of books on golf swings, the best way to defeat your opponent may be to fill his head with the concept of conservation of angular momentum. He may draw his cut so much that he misses you entirely. Changing your swing just to conform to a theory, which might be nothing more than a fad, isn't wise. If some part of your natural swing provides an opportunity to make use of this, then that is well and good. But if you end up with an overextended swing, just to provide room to draw the sword inwards, this theory is probably doing more harm than good. This information does seem in some way related to a draw cut; although most draw cut theory is based on slicing. Additionally, there seems to be some debate over whether an impact with a slight inward component of motion is a draw cut, or whether a draw cut should have the draw as the major component, with the impact of only secondary importance.

The conservation of angular momentum does cast doubt on push cuts, which is a popular concept with some in the fencing community. Some fencers have even held that Western swordsmanship uses push cuts, whereas Eastern swordsmanship uses pull cuts. As the historical manuals show, this is doubtful. In extending the blade outward during rotation, energy is drained from the blade, which is doing work to the arm during the arms extension. Additionally, in order to execute a push cut on a fast time scale, in the time of the hand, you have to swing without the arm reaching full extension prior to impact. This requires that the radius of the hand's arc be smaller than normal. As the previous section on natural rotation shows, decreasing the hand's arc also increases the angular distance that the hand must traverse before the sword naturally pivots around. This means a greater component of the swords rotation has to be provided by the forearm's torque. This begins to convert the swing into a wrist cut, but with decreased force due to the energy drain from the extension of the blade. An argument could thus be made that Eastern, and medieval, swordsmanship uses real cuts, and fencing, in some regard, doesn't.

Energy Transfer

Awkwardly worked.

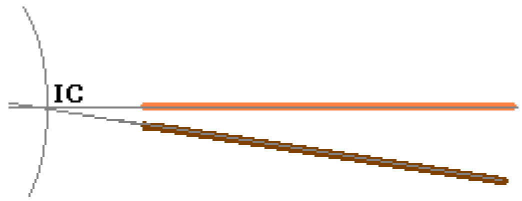
The percussion point does not determine how the sword's energy is transferred. That comes from the way the hand, sword, and target react to the applied forces. The distance from impact point to percussion point determines the ratio of the hand reaction force versus the target force. But the blow's mechanism is delivering impulse, which is force multiplied by time. Impulse also works out to be equivalent to the change in momentum, so we can say that the blow works by applying an impulse to the target, or increasing the targets momentum. However, if you really just wanted to add momentum to your target, just give it a push. The cut comes from delivering this impulse as a really short, sharp, spike of energy, which results in an inelastic collision. Given that the force on the target and the hand shock force are directly related, every bit of target force applied at any time during the strike brings about a hand force related by the above graph. These forces always occur in the given ratio and for the entire time duration of the strike, as long as the hand is treated as a fixed axis. The hand, under the action of the applied forces, due to missing the percussion point, will not remain fixed. However, if the hand mass is considered very small and movable then the sword will more easily move the hand, covering distance. Distance moved multiplied by the hand forces, when summed over the duration of the strike, would give the energy dumped into the hand. If the target doesn't yield, then the impact forces on the target have covered no distance, and delivered only a small amount of kinetic energy to the target. But we may have dissipated a great deal of the sword's initial kinetic energy into the target, in the form of mechanical deformations, which would be dents and cuts. All of this occurs while the highly movable hand has absorbed some of the sword's energy, even if the target forces where considerably larger than the hand forces. This is the energy you feel all through your arm when you strike at a particularly hard and unyielding target. Your sword and arm go "twang".

The Instantaneous Center of Rotation

From some of the many studies of baseball, we learn that the bat doesn't rotate around an axis located anywhere in the hands. It rotates around a point located 0.15 to 0.20 meters inside the knob (The knob is the bat's version of a pommel).

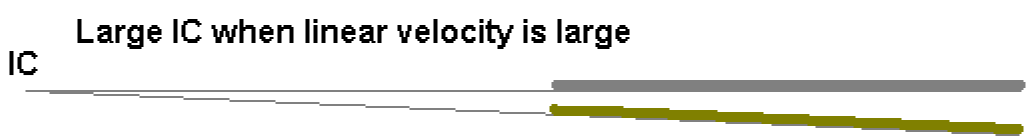
This point is termed the instantaneous center of rotation, or IC, and arises because of the way a bat is swung. Two handed sword cuts, at least horizontal ones, should be very similar. However, swords are much more varied in their application, with a wider variety of stances and target motions. But let's begin with the standard baseball swing.

Here is a little drawing of the path of a stick showing the IC.

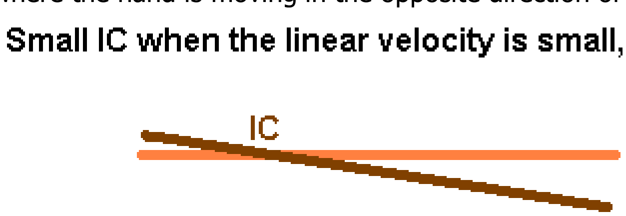


The instantaneous center of rotation comes into play because your arms are rotating around your body, while the weapon is rotating around your wrists. This ends up making a pretty complex path to analyze, but we only care about the path at impact. If you have a high-speed camera system, you just draw a line through the weapon on two successive frames. Where these lines intersect is the instantaneous center of motion, or IC. The angle formed by the two lines in the two successive frames divided by the camera's frame rate gives you how fast the object is rotating (ω). If you want to know the weapons kinetic energy, just calculate the moment of inertia about the IC using the parallel axis theorem, where y is the distance from the IC to the center of mass, so $I_y = I_{COM} + my^2$. The rotational kinetic energy is just $KE = \frac{1}{2} I_y \omega^2$. Note that from a fixed perspective, the center of rotation isn't in motion, since your camera wasn't moving along some path. The camera stays fixed, and merely records two images of the weapon in motion. The center of rotation doesn't travel along with your hand, although a series of photos will show the IC location develop and move over time, throughout the swing.

If you increase your blades linear speed relative to the target, such as striking objects to your side, while running, the IC becomes very distant from your hands. This happens because your linear velocity is starting to dominate. Imagine two snapshots of a strike from a moving car. The blade may only rotate by 1 degree in between frames, but in between taking the two blade images the sword might have covered a large distance over the road. The lines made by the two blade images intersect very far from the blades.



Conversely, if you are moving your hands backwards during the strike, the IC moves close to, or even overlaps, the blade. This would also be a good description of the rotation of a wrist cut where the hand is moving in the opposite direction of the blade tip.



Now the question arises as to why we don't strike with the percussion point relative to this instantaneous center of rotation. Striking this point against a hard target would bring the weapon to a complete stop, with no energy remaining in residual rotations. The answer is complex, and mainly concerns the fact that this part of the blade is moving slowly, which tends to accelerate the target, but not cut it as well. However, against a very heavy target this may be the best, though also the most painful, approach.

My Earlier Thoughts on Hand Shock

I used to think that since such an impact would also have to stop your arms, using up some of the weapon's energy, it would have less energy left to apply to the target. I discarded this notion once my later impact simulations said otherwise. But since this thought may have occurred in some form to period swordsmen, especially since it's arrived at by analogy and logic, well known in the Renaissance, I'll explain it. The basic idea is that the hand shock must represent energy that the weapon didn't deliver to the target, and since the weapon only had a finite amount of energy to start with, hand shock must decrease the shock felt by the target. It would be easy to arrive at a notion that there's only so much *shock* in the blade, and you don't want to drain any of it out through the handle. It turns out that this isn't the way things work, and hand shock merely means that while the blade was yielding way during the impact, your hand stiffened its resolve, though painfully so. The effect on the target is very small, though, and for a single-handed strike hardly amounts to that of a cut delivered by a newborn baby. But this is only if the shock occurs while the cut is still in progress. If you make a sharp impact that's over before your hand slams into the handle, then you get all the pain, but none of the gain. But I certainly don't yet have any confirmation that anyone but me actually thought of it this way.

However, you can avoid all these strange and sometimes painful energy transfers by merely striking with the percussion point relative to your hand. Unfortunately the impact only has one percussion point, so it can't cover both hands. But I'll stipulate that the strike is with the percussion point relative to your upper hand, and not worry over the lower hand. By striking with the percussion point relative to your hand, instead of with the percussion point relative to the center of rotation, you keep your hands from affecting the energy delivered by the blow. Looking at the baseball bat, it's pretty apparent that the best method of using the tool is to strike with the percussion point relative to your hands, since that's pretty much the location that sends baseballs go flying over the fence. However, the bat is also designed to produce this result, and a random piece of wood won't necessarily produce the best hits at the hand's percussion point. That's why you don't interchange baseball bats with softball bats.

Consider that when you strike a blow, there are only a few places for the energy to go. It could travel into the target, it could stay in the blade, as either residual rotation or vibration, or it could go into your hand. If you strike with the percussion point relative to your hand, then your hand remains the as axis of rotation of the new motion, relative to the motions existing prior to impact. Another way of stating this is that your hand receives no torque from the impact. Since this also means that the grip of the sword receives no torque, its velocity does not increase or decrease. The end result of this is, that if you let go of the grip in the instant prior to impact, forming your index finger and thumb in a circle about the handle, yet not touching the handle, the impact will not cause the handle to touch you. Essentially, by striking with the percussion point relative to your hand, you are not, in essence, touching the sword. You've pulled yourself out of the impact equation, and thus can't affect the force of the impact.

To illustrate that the effect of hand shock can act to increase the blow felt by the target, I'll present a little exercise. When you strike inside the percussion point, the handle kicks into your palm. By action and reaction, this is the same as your hand slamming into the handle. Try this experiment with your sparing partner, to show how weak this effect is. Have your partner place your blade against his leg, just inside the percussion point relative to your normal hand location, while holding it with the handle toward you, just as it would be mid-way through an impact. Now, take your hand and hit the handle as hard as you can. When I tried this, repeatedly, in all sorts of configurations, the "target" felt almost nothing, but the person striking the handle ended up with nearly bruised hands. If you back calculate the percussion point equations, your partner is reasonably close to the percussion point, which becomes the axis of rotation. Basically you've reversed positions, and the closer your partner is to a clean hit, the less your "hand impact" affects him. At the percussion point, not only does your hand not impact the handle, any such impact would impart absolutely no extra force to the target. If you strike inside the percussion point, you will feel some kick back into your palm. But the corresponding reaction of your palm kicking the handle, will only impart a small percentage of this force to the target.

Chapter 5

Applications of Percussion Point Theory

Let's start with the basics, the stick. Sticks and clubs are some of the most basic weapon swung by hand, and familiarity with them is essential for understanding the design and behavior of more advanced weapons.

Assume you've got a basic stick, or staff, with uniform density and cross sectional area, so mass is linear with length. The center of mass (COM) is right in the middle, so no fancy balancing is required to find it. You can easily measure the weight with a scale, or get an idea of it with your hand. You can measure the length, or just eyeball it. We've covered the moment of inertia, and you can measure it for the stick or use the textbook formula to come up with it, since long, thin, uniform objects are well studied ($I_{COM} = \frac{mL^2}{12}$). So where is the percussion point of a stick or staff?

Here we have a staff with two sets of axis/percussion point pairs marked on it. But to get them I had to do some math. I'll soon show how you can skip the math, and find the pairs in about two seconds by hand. But first, let's go through the analysis. It builds character.

First, note that the mass of the staff goes up uniformly with length, as this is a non-tapered staff. How much the mass goes up, depends on the diameter and density of the wood, but fortunately we don't care about that when studying the properties of percussion points. Those things just make the staff stiffer and heavier, but don't affect the geometry of this study.

So, mass m is some constant k times the length L, or $m = kL$. The moment of inertia of the stick about its center of mass, by the textbook formula for a long thin object is

$$I_{COM} = \frac{mL^2}{12}$$ **Moment of inertia for a uniform rod**

But substituting in the formula for the mass ($m = kL$) we get $I_{COM} = \frac{kL^3}{12}$.

The percussion point distance from the COM, relative to some axis of rotation (measured from the COM) is $Q = \frac{I_{COM}}{m \cdot x}$ or $Q \cdot x = \frac{I_{COM}}{m}$. Substituting in the formula for I_{COM} we get

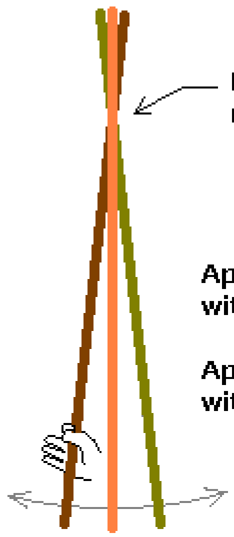
$Q \cdot x = \frac{k \cdot L^3}{12 \cdot k \cdot L}$, or

$$Q \cdot x = \frac{L^2}{12}$$ **Percussion point formula for a uniform rod**

This lets us calculate the percussion point for any axis of rotation we choose.

Hold the stick by one end. Then $x = \frac{L}{2}$, the percussion point is $Q = \frac{L}{6}$ from the COM, or $\frac{1}{3}L$ from the other end. These are the red circles in the drawing. Get a stick or steel rod, and try hitting this point with a hammer. Your hand, at the end, feels no shock. If you hit elsewhere, you start picking up impact force in your hand. If you move your hand elsewhere on the stick then the percussion point, relative to your hand, also changes. If you hold the staff at $\frac{1}{4}L$ from one end, then your hand's percussion point is located at $\frac{5}{6}L$, or $\frac{1}{6}L$ from the other end. These points are the green circles in the drawing. Try hitting these till you see what's going on. A simple rule applies to the staff. To make the last 1/6 of an untapered staff deliver sharp impacts hold the area that's in between one quarter and one third from the butt end. The one-third position has a percussion point at the tip. The one-quarter position has a percussion point 1/6 back from the tip. The percussion points form a distorted map of the grip, with the foremost grip position having it's percussion point farther out than the rearmost grip position. Since the percussion point moves further out, as your grip gets closer to the center of mass, and is already at the tip at the 1/3 position of the shaft, what happens when you move inside the 1/3 position, toward the center of mass? The percussion point moves right off the tip! An untapered staff has no physical percussion point relative to any grip position in the middle third of the shaft. How can this be? No percussion point on a staff? Yes, it's true. Keep in mind that the percussion point exists relative to some axis of rotation, and that the axis of rotation and the percussion point are completely interchangeable. Looking at these reversed positions, apply force inside the 1/3 length zone, and observe the corresponding axis of rotation, which is the center of percussion. We can directly observe this behavior. As you move toward the center of mass, the staff rotates less for the amount of sideways motion you impart. It begins to rotate about some axis located far away. If you apply force directly to the center of mass, then the axis of rotation is infinitely far away. This also means that to rotate an object around the center of mass, would take an impact infinitely far away. Such impacts don't happen to real objects. But this is the quickest way to find the percussion point. It is the test a child can do in two seconds. To find the percussion point relative to your hand, apply a rapid move to your hand and observe the corresponding axis of rotation. The location of this axis of rotation is the percussion point relative to your hand.

Pivot Test for COP



This test is extremely quick, and fairly accurate. The greater the inertia of the object, the easier the test, as the small involuntary torque from your hand has less affect on big objects. The test does have an error, due to the object tipping over a bit, but the test's ease more than compensates for its slight imprecision. With it, you can find the percussion point of pole-arms very quickly. Almost as quickly as you slide your grip on the shaft, up and back. On swords I can get a repeatability and accuracy within an inch, usually even a centimeter. To do this, I rest the sword's cross-guard across the top of my hand, and only let my thumb and forefinger touch the handle. Then I shift the hilt sideways, only allowing my thumb and forefinger knuckles to apply any force. Shift the hilt once to the right, then return. Keep your eyes focused on the blade, and zero in on the pivot point. This should be almost directly in front of your face. Repeat the test as necessary, till your pretty confident in the repeatability of the percussion point location.

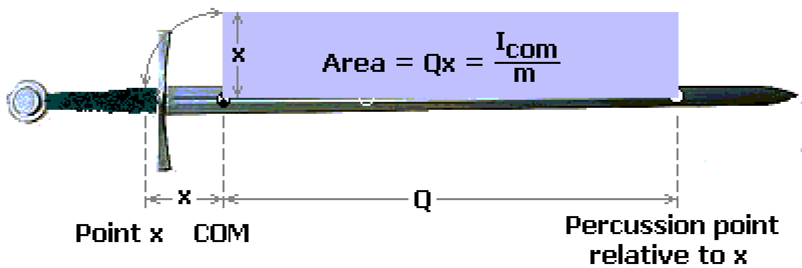
Once you've practiced it, the test can be done in the normal course of getting a weapon's feel. If your good at it, no one will even see you do it. It's the magic test that no one knows about. It's also enlightening to perform the converse of the simple sword test. Find the percussion point relative to your forefinger, using the normal percussion point test. Then take hold of the percussion point, located out on the blade, and repeat the test with the sword help by the blade, upside down. The sword will now pivot about the hilt, very near the cross-guard. When you do this, you're acting like an impacting target, in the sword's frame of reference. Note that you, as a target at the percussion point, are not moving the hilt location. From this percussion point location, the application of external force (not a moment), cannot apply a torque to the hilt, and thus can't kick the hand.

There is another striking thing about the percussion point, moment of inertia, mass, and center of mass location. They are all interrelated by the by now familiar equation for the percussion point $Q = \frac{I_{COM}}{m \cdot x}$, or $Q \cdot x = \frac{I_{COM}}{m}$, if you prefer. The one thing you shouldn't overlook about this equation is that it's an equation! If you measure any three of the parameters, then the forth is automatically determined. Measuring the mass is trivial, then and now, as is measuring the balance point to find the value x. You can get a feel for the moment of inertia by just waggling the sword, but for a sword smith this might be a bit crude. It may be a way to judge blades as they come in the door, but not a good final check of a weapon. Given how easy it is to measure the percussion point, by applying a single force to the handle, and that there are much more accurate ways of doing this test that aren't much more complicated, the moment of inertia needn't have been measured to be controlled.

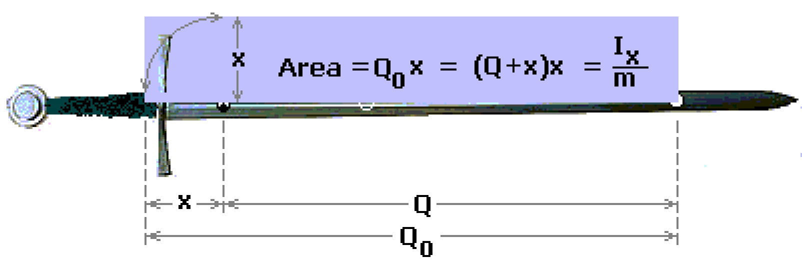
If your percussion point measure is off by a centimeter, it will only affect the results by about 4% (for a .60 meter value of Q). This is far better than the variation in moment of inertia that's found in current reproduction swords. But this also gives us a nice little bit of knowledge. On a reproduction of a particular sword, of the four critical parameters of percussion point, moment of inertia, mass, and balance point, it is physically impossible to have only one of these parameters wrong. Either all are correct, or two or more are in error. This is pretty profound stuff. If percussion point location is in fact vital to a sword, then given a replica's mass and inertia, which determines its feel, there is not an option of getting the balance point location that you prefer. It's a given part of the design. When you live in the word of bad reproductions and limited knowledge, it can be hard to know why exactly you prefer particular sword properties. Some swords feel good and work well, others don't. Now we're finding out why.

Visualizing the Feel

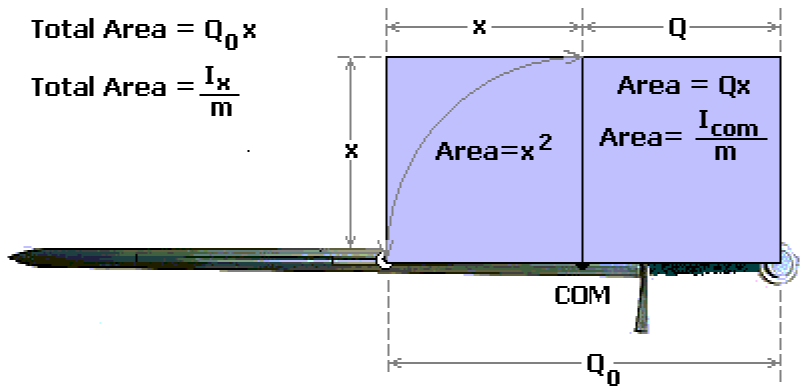
To help illuminate all this, here are some pictures that geometrically show what percussion points, and inertia to mass ratios, and all the rest, mean.



Here, we can see that $Q \cdot x$ gives the moment of inertia to mass ratio, taken about the sword's center of mass. But x is just the distance from your hand to the sword's center of mass, and Q is the distance from the sword's center of mass to the hand's percussion point. Multiplied together, these define an area, as shown in the diagram. This area is the sword's moment of inertia to mass ratio, which is constant. No matter where you move x, your hand location, the corresponding move by Q, the percussion point relative to your hand, acts to keep this area completely unchanged. If x approaches zero, then Q goes to infinity.



Here is a similar diagram, but this time also showing the moment of inertia about point x, instead of the moment of inertia about the sword's center of gravity. Note that a little square has been added, which is of length x and height x. Since the mass remains constant, unless we add or subtract material from the sword. This means that the ratio of the moment of inertia about x, versus the mass of the sword, increases by x^2, no matter where we move x. This is logical, since if we multiply everything by the mass, so that we are looking at the moment of inertia, itself, then the inertia goes up with mx^2, in complete agreement with the parallel axis theorem. For a good illustration of this, lets flip the sword around, and use it for a murder stroke.



Here is the diagram of the murder stroke. As you can see, the area of the rectangle, between the center of mass and the pommel, is the same as the area in the above diagrams. This area is constant, and allows you to find any point's corresponding percussion point. Unfortunately, it can be hard to accurately do this by eye. I doubt a knight walked around with areas and volumes dancing in his head. However, the method is still useful. In this drawing, you can see that the additional area, due to the distance from the hand to the center of mass, is getting quite large. If you held the sword by the tip, the resulting square would be far larger than the rectangle representing the sword's inertia to mass ratio, considered about its center of mass.

We can also illustrate the ratio of hand impact forces to target impact forces, just be going back to our earlier equations for this ratio.

Another Insight Into Feel

Another way of looking at the percussion point equation is by looking closely at the denominator in either the equation $Q = \frac{I_{COM}}{m \cdot x}$, or $Q + x = \frac{I_X}{m \cdot x}$. In both of these equations, the denominator is simply the mass multiplied by the distance from your hand to the center of mass, or balance point. When you hold a weapon horizontally, gravity pulls down on the weapon as if it were solely acting on the weapon's mass and center of mass location. To counteract this force, you have to apply an upward force, to keep the weapon from falling to the floor, and a static torque, to keep the blade from dropping. If you don't apply a torque, and simply provide an upward force, then your hand could be replaced by a string, which would merely cause the weapon to start swinging blade down, like a pendulum. The amount of torque you have to provide is simply $\Gamma_{STATIC} = g \cdot m \cdot x$, where g is the acceleration due to gravity. Using this, the percussion point equation becomes $Q + x = \frac{I_X}{\Gamma_{STATIC} / g}$. Since the acceleration of gravity is constant, unless you're a Space Marine or Jedi Knight, we can ignore the g in the equations, and just say that the percussion point distance is a linear function of inertia over static torque. The percussion point distance, relative to wherever you hold an object, is a linear function of the moment of inertia your hand feels when maneuvering, divided by the object's static moment, which is the torque that you feel when holding it horizontally. If you move a blades balance point back, to make it "feel" better, the denominator in this equation gets very small, and your hand's percussion point goes shooting way out. Conversely, if you don't do anything to balance the blade, the static moment stays very large, and the percussion point moves back in.

But the point to all this is that you can directly feel static moment, and intuitively grasp the meaning of it. By maneuvering a sword you can get an intuitive estimate of its moment of inertia about you hand, and by just holding the blade you automatically get a measure of its static moment. If a blade is hard to maneuver, but feels "balanced", then you already know the percussion point distance is very large. You should immediately be suspicious of it, since the percussion point may be way past the end of the blade. If a blade maneuvers well, but feels horribly balanced, the percussion point distance is probably quite small. You are holding a saber, or possibly the heavy end of a tapered shaft, and the percussion point is probably between ½ to 1/3 back from the blade's tip.

The moment of inertia about the forefinger, I_X, which is a good indicator of the dynamic feel of the sword, turns out to be just $I_X = \frac{(Q+x)\cdot\Gamma_{STATIC}}{g}$. So in weapons where the percussion point is at a constant distance from you hand, the moment of inertia is a direct, linear function of the static moment. If you know where a weapons percussion point is, just mentally multiply that by the static moment, and you get a measure of the weapons moment of inertia, or lack of maneuverability. If you move your hand more toward the center of mass, to improve the static moment, you've also driven up the percussion point distance. But by the parallel axis theorem the inertia about your hand has lessened a bit, so the increase in percussion point distance doesn't exactly correspond to the improvement in static moment.

Mass Impact Weapons

The percussion point shows up in the design of many other things. Baseball bats and tennis racquets have to have very precise control of it. Sports equipment fortunes have been made from correctly understanding the percussion point and associated nodes of oscillation, the physics of hand swung tools. But even in the very practical world of farm implements, the percussion point comes into play. Since hitting the sweet spot means better cutting and less hand shock, you'll find that even an axe has a precisely controlled percussion point. I went out and bought a top selling axe, just to measure its physical parameters (then gave it to my boss, as I had no other use for an axe).

Standard 3 ½ Pound Axe

The axe is a Collins Commander, a 3 ½-lb model, which weighs 4-lbs, 10.5oz. The 3½-lb description is a measure of head weight.

The edge is 4 5/8" long, and sweeps up (the upper beard) 3/8".
The center of mass from the top of the head (discounting the upper beard) is 5.875".
The distance from the top of the head to a line centered on the edge is 1.9375"
The distance from center edge to the center of mass is 3.9375", or 0.328125 feet.
The moment of inertia about the center of mass is a tiny 3.24218 lb-ft^2

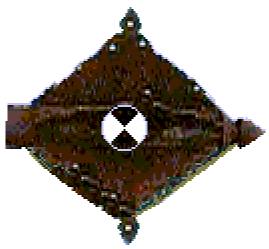
The Axe's Percussion Point

Assuming that the percussion point would be the exact center of the blade edge, as all cutting forces (the shock) will be centered on this point, we can calculate the resulting axis of rotation. This point came out to be ½" from the fattest part of the handle, as pictured. This is also where the pinky finger of the lower hand holds tight during a swing, to keep the axe from flying out of your hands. This seems to be the point chosen to receive the least shock during impact. I don't know whether this is point was chosen by years in the marketplace (Collins has been making axes since 1829), or by design. However, if you replicate that handle with another material, such as fiberglass, and don't redesign the geometry, then the center of mass will move and the new handle will sting your hands.

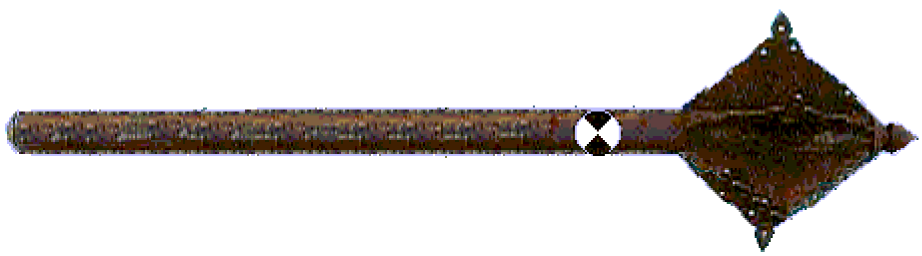
As long as we're looking at the axe we might as well mention its moment of inertia. Moving back up the handle about five inches is a point in between the hands. The moment of inertia at this point is 18.19 lb-ft^2 (0.768 kg-m^2). So, if you tried to swing an axe like a sword, its inertia would be very high, needless to say.

But this also points up an interesting fact. In an axe, the percussion point does not in any way coincide with the center of mass. Most people would guess that an axe's center of percussion would be in the middle of the edge. But they would do this by incorrectly assuming that the center is aligned with the balance point. It's not. It's just the center of the big heavy part. The same assumptions would apply to a mace.

When you look at a mace, you might think that the center of mass lines up with the sharp points, but in fact, it doesn't. If I cut off the handle, then we would have a mace head with a center of mass located in line with the points, but obviously, the handle moves the center of mass some distance down the shaft.



So instead a mace would balance something like the following.



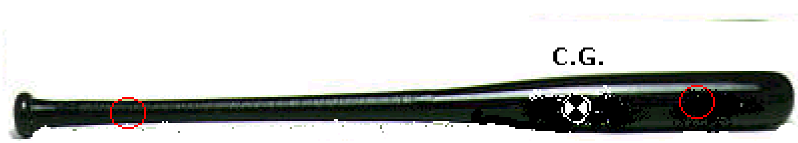
For an axe or mace, the percussion point does not coincide with the center of mass, nor does the center of mass coincide with the center of the big heavy part of the weapon. If the weapon is well designed, then the percussion point will coincide with the location of the edge or spikes. This raises an obvious potential problem with mace and war axe reproductions, which one would think would be idiot-proof designs. Sadly, this is not so. It's possible to screw up the design of a heavy headed impact weapon. The result may not be as bad as screwing up a sword design, but the potential is there. So even with the correct weight, the balance point relative to the handle and the head, along with the weapons moment of inertia, still determine the design's merit. But for all the caveman jokes we make, who would've thought somebody could screw up a club?

Another item to note is that in a sword the distance x, from the hand to the center of mass, is smaller than the distance Q, from the center of mass to the hand's percussion point. So the set of percussion points relative to your hand is pretty broad. You can strike a fairly wide range of points on the edge, and still have the corresponding axis of rotation somewhere in your hand. But with a mass weapon like an axe, x is quite large while Q is small. So only a small area around the weapon's designed impact point has a corresponding axis of rotation in your hand. This implies that in some way mass weapons, such as axes, are more sensitive to the striking point than swords. On the other hand, for a constant given miss distance, the axe and sword will

have the same percentage of impact force kicked back into the hand. For the Collin's axe, hitting with the edge's tip will mean your hands feel 8.2% of force of the impact. This is exactly the same percentage as a sword with an equivalent percussion point distance. However, the axe is delivered a much larger force, so maybe this 8.2% is much more painful with an axe.
So axes are designed so they don't sting the hand when the blow strikes the percussion point. Interestingly, this is also why an axe must have a curved edge. With a straight edge you will usually hit with one end of the edge or the other, yet both points can't be a percussion point relative to a single part of your hand. Thus you get stung, with the handle kicking either forward or back. You can't design around this problem very easily, since if you decide to always strike with the far tip, then during chopping you end up aligning the notch in the log with the edge. Very quickly you start hitting with either one end of the edge or the other, leaving the percussion point problem unsolved. In the lucky case of a strike in between the ends of the edge, you've probably struck in exact alignment to the notch you've formed. The axe might just bounce off, because the pressure is spread evenly along the entire edge, and if the previous blow has undercut the wood, the wood can act like a spring. A curved axe attacks any such spring unevenly, cutting through it starting in the middle and then moving to the ends. That way all the little wooden springs aren't able to resist as one. It becomes obvious that a straight edged axe will either string your hands or bounce off, only cutting well on the first few chops at a log, before a notch has been formed. In a culture where everyone has to cut logs for heating and cooking, which encompasses any pre-modern one found in temperate forest regions, everyone would probably have such a notion. An axe shouldn't have a curved edge.

Curiously, a hatchet generally doesn't need a curved edge, unless it is used to cut large limbs and small trees, as if it were a small axe. In cutting and trimming smaller limbs, which is the usual use for a hatchet, a deep and even notch is generally not formed. As the hatchet usually strikes a fresh limb, the impact point is determined by where the first contact with the round limb is made. Even if the hatchet has a straight edge, it should make first contact in the center of the edge. This contact won't be even all along the edge. It will be focused at the closest part of the round limb, so the pressure at the impact point will be high. It will not be spread evenly all along the length of the edge, which tends to cause a penetration failure. To see these penetration failures in a straight edged hatchet, which are commonly available at hardware stores, strike hard, keeping the edge dead flush with a large flat piece of lumber. Most hatchets are used on small limbs, where the impact point on the limb is always centered on edge, which is larger than the limb. Weapons that make a fresh cut with each strike on a non-flat surface probably don't need edge curvature. This also includes swords.

Speaking of clubs, I might as well be thorough and show a baseball bat.



It shows the same behavior as the axe and mace. The percussion point relative to the handle is out past the center of mass (here labeled C.G., for center of gravity). Again, the center of gravity is not the sweet spot. Hits on the center of gravity will cause the handle to kick backwards into the wrists. But another interesting relation, which baseball research is developing, is that the sweet spot doesn't correspond exactly with the percussion point. Ideally it probably should, and on a baseball bat these points are close together, but not exactly the same. In baseball theory the percussion point is used to describe the smoothest impact point of a perfectly stiff bat. If the wood didn't flex, this would be the true sweet spot. But the impact also sets up some very large, and violent, oscillations in the bat. The sweet spot, in their thinking, is a convolution of the stiff body impact dynamics and the flexible body's oscillations. But even these don't always exactly correlate with the point on the bat that gives the highest ball speed, which is an interesting subject that we will take up in another chapter.

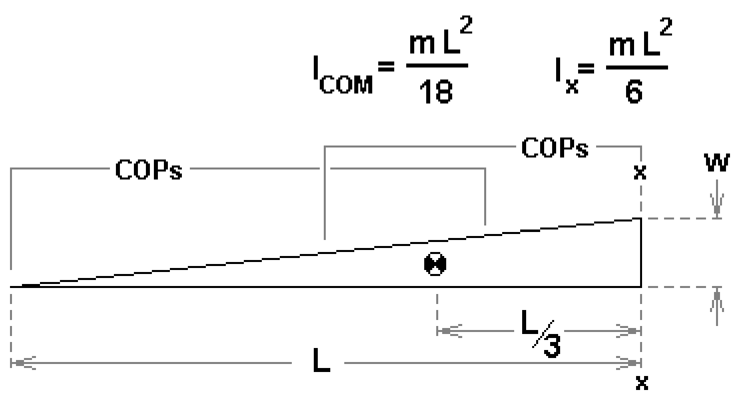
Percussion Points of Weapons with Tapered Mass

We've covered the staff in some detail, but what happens if you taper the ends? If you held the mass constant, then the middle would be thick and the ends somewhat thin. This would look somewhat like a longbow, and gives you faster maneuvering. The reason that the tapered staff maneuvers faster is that I_{COM} is quite a bit less, depending on the taper ratio. But what about the percussion point equation $Q \cdot x = \frac{I_{COM}}{m}$? If we've held the mass constant and reduced the moment of inertia, by rearranging the location of the mass, overall, then the tapered $Q \cdot x$ values must be smaller than the $Q \cdot x$ values for a staff of uniform thickness. Even if we've trimmed the mass, instead of holding it constant, the remaining mass gets concentrated in regions where x will be small in the inertia equation

$$I = \sum_{i=1}^{n} m_i \cdot x_i^2$$

So the moment of inertia to mass ratio will be smaller. We thus end up with a pretty useful bit of knowledge. Whenever you taper a weapon, the percussion points move toward the center of mass. So if we've got a tapered staff, the simple formula of holding 1/3 back from one end doesn't apply anymore. We actually have to either measure the moment of inertia, and come up with a whole new set of $Q \cdot x$ percussion point pairs, or do the simple little rotation test. Pretty obviously the simple rotation test is the way to go, unless your answering museum type questions.

Let's look at the moment of inertia of a uniformly tapering object, which is easy to calculate.



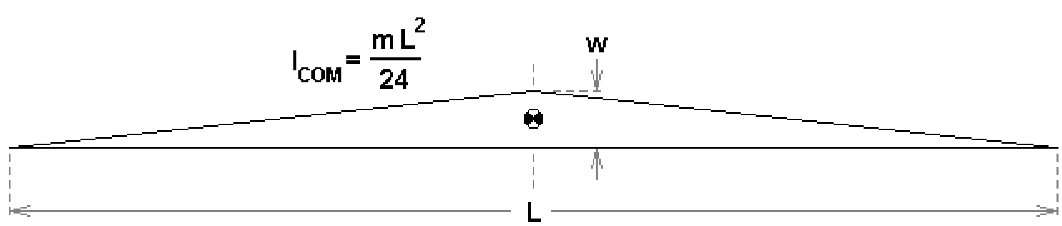
Recall that the moment of inertia of a uniform stick was $I_{COM} = \frac{mL^2}{12}$. With uniform taper we get $I_{COM} = \frac{mL^2}{18}$, or only two thirds as much. Swinging the tapered weapon from the heavy end gives as a moment of inertia of only $I_{END} = \frac{mL^2}{6}$, only half as much as the uniform rod, which gave $I_{END} = \frac{mL^2}{3}$. The moment of inertia calculated at the light end of the weapon is obviously higher than a straight weapon, and is given as $I_{LIGHT\_END} = \frac{mL^2}{2}$, which is 50% more than the straight weapon. This is also three times higher than the moment of inertia about the heavy end of the tapered weapon.

The distance from the end to the end's percussion point can be found by the standard equations.

$Q \cdot x = \frac{L^2}{18}$ **Percussion point formula for a uniformly tapered rod**

If we hold the weapon by the heavy end, then $x = \frac{1}{3}L$, so we get $Q = \frac{1}{6}L$, and $Q + x = \frac{L}{2}$.

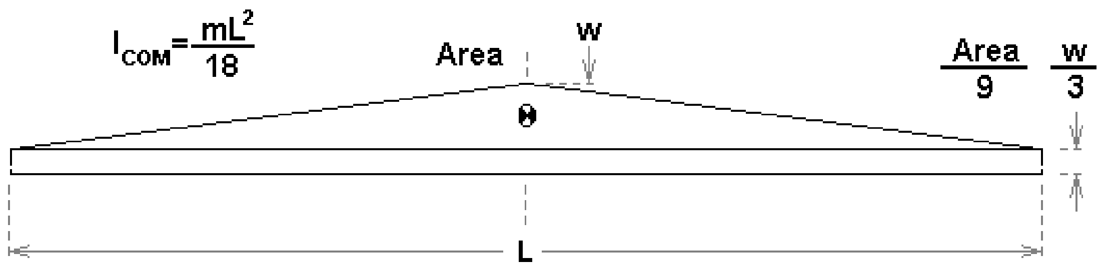
If we hold the light end, then $x = \frac{2}{3}L$, so $Q = \frac{1}{12}L$, and $Q + x = \frac{3}{4}L$. The percussion point of a uniformly and completely tapering weapon, relative to the heavy end, is exactly in the weapon's center. Relative to the light end, the percussion point is exactly ¾ of the weapon's length. This also tells us that to make the light end of the weapon a percussion point, we must place our hand a fourth of the way up the blade.



If we uniformly taper a staff, we get only half the moment of inertia of a straight rod, if we have the same mass.

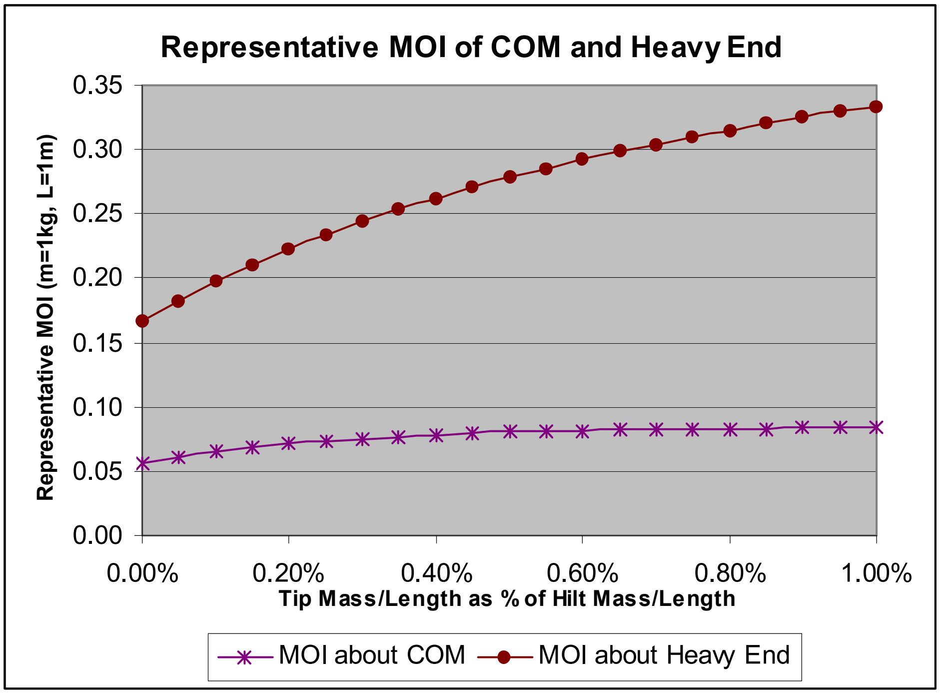
$Q \cdot x = \frac{L^2}{24}$ **Percussion point formula for a double tapered staffed**

If we hold this staff by the end, then $x = \frac{L}{2}$, $Q = \frac{L}{12}$, and $Q + x = \frac{7}{12}L$. So the percussion point is just past the middle. This also means that if hold the staff very close to the middle, exactly $\frac{1}{12}L$ from the center, your percussion point becomes the far end of the staff. So not only is the tapered staff twice as maneuverable from the center as was the straight version, it also has tips that nearly become percussion points when half staffing. However, keep in mind that the percussion point is always on the opposite side of the center of mass as its mate. When half staffing we are probably striking the blow with the hand that's toward the target, which has a percussion point on the wrong end of the staff. But for quarter staffing, having the forward hand's percussion point near the tip becomes a definite probability. This comes about because we don't really taper a weapon out to nothing. Whether a sword blade or staff, we don't end in a thin spike or razor blade. So real mass profiles would more likely be represented by a marriage of a uniform rod coupled to a tapered one. Suppose we had a staff where the mass profile is tapered uniformly to the tip, but where the tip thickness was only one third of the center thickness. This would mean the tips cross sectional area, and thus mass, would be only a ninth of the area and mass in the staff's center. So you might have a staff with a width of 1.5 inches in the center, and half-inch diameter ends.



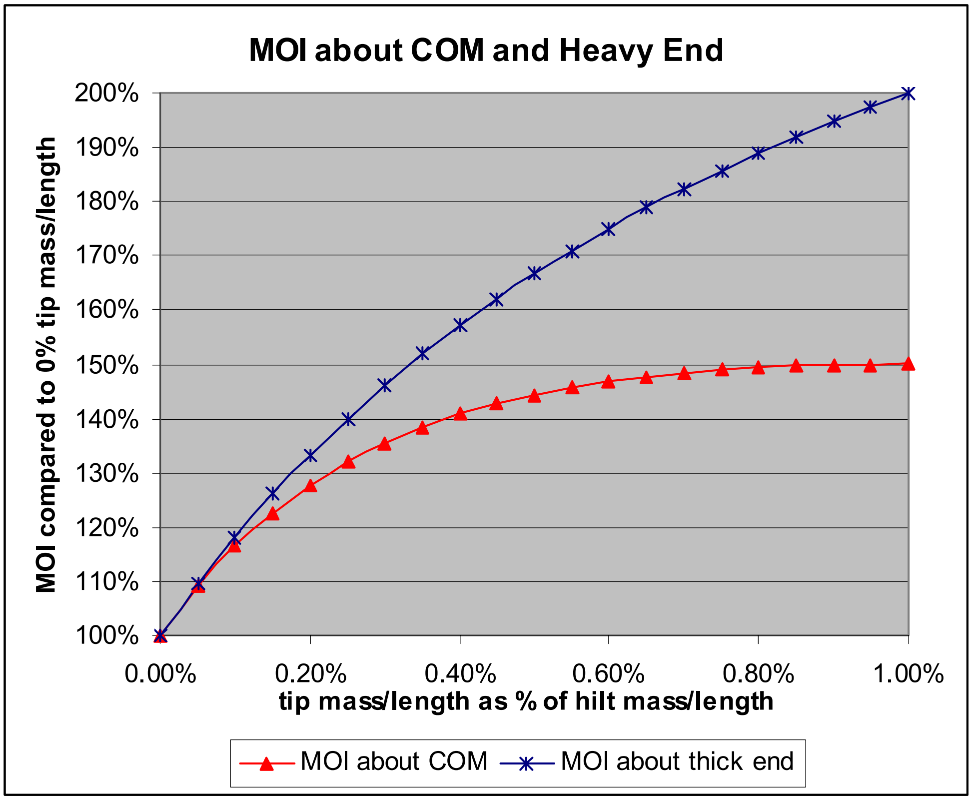
This staff is midway between a non-tapered version, with $I_{COM} = \frac{mL^2}{12}$, and the fully tapered version, with $I_{COM} = \frac{mL^2}{24}$. So the reasonably tapered version only has 66% of the moment of inertia about the center of mass that the straight staff has. The percussion point relative to the end is 11/18 of the staff's length, or 7/18 of the length, when measured from the other end. This means you can place your hand in the area of 40% of the staff's length, and strike with the tip. You could also back down to the 1/3 point, and strike with a point 1/6 of the length from the tip.

Let's also take a quick look back at the sword blade model. A tapering mass combined with some non-tapered component is a better description of a blade. A sword tip doesn't resemble an X-acto knife. If we make a blade with a mass that tapers uniformly to the tip, where it retains only 25% of the mass at the hilt, then the balance point is 40% of the length from the hilt, and the moment of inertia will be $I_{COM} = \frac{mL^2}{13.636}$. This is still 88% of the moment of inertia of a straight, non-tapered blade, and 132% of the moment of inertia of the purely tapered blade. However, the moment of inertia about the end is also not much different than the non-tapered blade. As stated before, just a little bit of extra mass near the tip can really impair the maneuverability of a sword. In rough numbers, as you increase tip mass/length relative to the hilt, the hilt being quite thick, the maneuverability plummets, as shown in the following graphs.

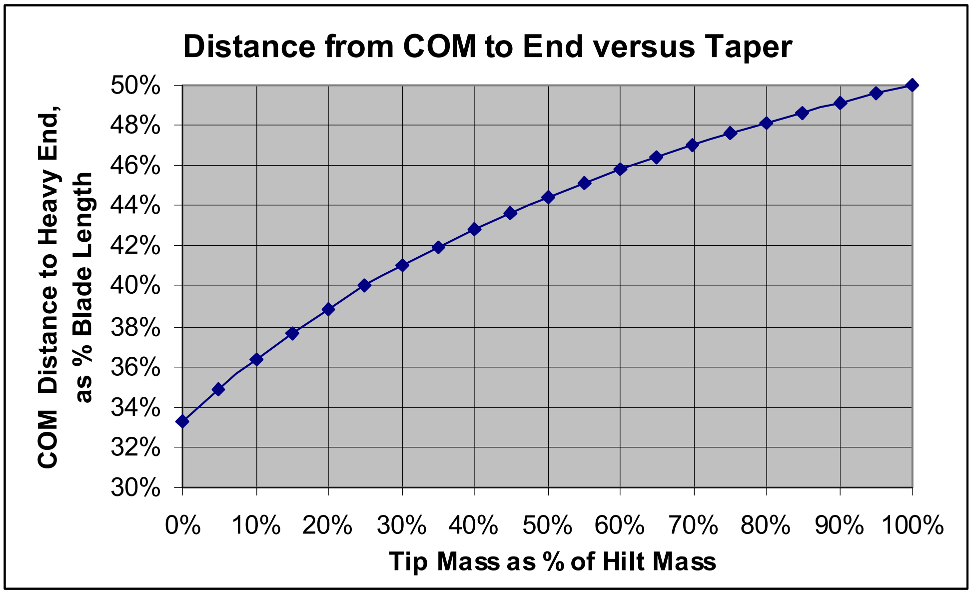


Above is a graph showing how the moment of inertia increases as you make the tip mass a greater percentage of the hilt mass, yet with uniform mass tapering. You can see that the moment of inertia about the blade's center of mass increases somewhat, but this increase starts to level out. This is because the added mass is moving the blade's center of mass closer and closer to center, and the taper becomes less significant as the increasing amount of tip mass makes the blade more and more like a straight rod. However, the moment of inertia about the end keeps increasing at a significant rate. This is because the small amounts of additional mass are being added very far from the end being used as the axis of rotation.

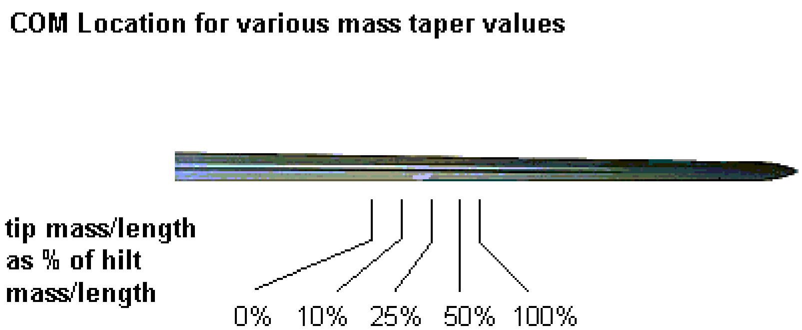
This points up that changes to the sword's moment of inertia, considered about the hand, may be affected to a much larger degree than the sword's moment of inertia about the center of mass.



This graph shows the same effect, except the units have changed to reflect how much the blade differs from one that has pure mass taper, meaning a blade that tapers to nothing at the tip. Here the moment of inertia about the center of mass looks like it makes a steeper climb, but this is due to the change in scale. Here again, we see that the moment of inertia about the heavy end shows a much more dramatic sensitivity to tip mass, as compared to hilt mass. As stated before, mass around the hilt doesn't affect a sword's performance nearly as much as tip mass. Additionally, extra hilt mass, or mass back in the strong part of the blade, makes the sword much stiffer during defensive use of the sword's flat, meaning much less blade wobble when the opponent's edge hits your flat. Considering that this extra mass in this area carries defensive benefits and little performance penalty, you can see why many period swords have very thick blades in this area.



We also have the center of mass moving away from the heavy end, as the amount of taper decreases, as shown in this chart. If you have a bare blade, its balance point can give a fair indication of how much mass tapering it has. This mass tapering can be easier to feel, than to see. But this chart will help in evaluating what kind of blade you might have in your hand. It ignores the effect of the tang, which will be addresses in a different section on tang thickness and strength.



Unfortunately, the blades with the best mass tapering also have the worst percussion point locations, relative to the heavy end of the blades. A blade with pure mass tapering, meaning the tip fades to nothing, has a natural percussion point only halfway down the blade.

In conclusion, since all our decently crafted swords taper, especially in thickness, from hilt to tip, this will move the swords percussion point pretty far back on the blade. Wouldn't it be wise to come up with a way to move the percussion points back out to the tips, while retaining the blade taper? (I consider the axis of rotation and it's corresponding percussion point as the same thing, since they're interchangeable). Why have a long blade, if impacts using the business end results in hand shock and weak impacts? Well, aside from moving the mass out to the tips, and making the sword rotate like a barbell, with a fantastically high and unwieldy moment of inertia, is there any possible way to do this? Yes, it turns out there is an incredibly clever way to do it. A method so clever that best I can tell, unless people are keeping secrets, it's gone right over the heads of all 19th and 20th century researchers.

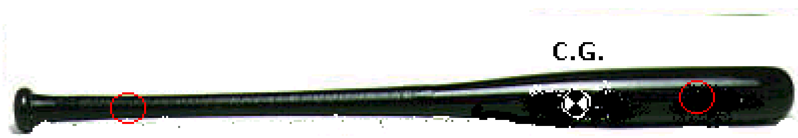
Take a look at the axe again. If you held it at the edge, then the corresponding percussion point is indeed out at the other end. In fact, both percussion points are within an inch or two of the ends! The baseball bat shows the same thing, though not to quite the same degree. The percussion points have moved toward the ends. Suppose the percussion point on the right was he grip, and the left was the tip. Just turn a club around and we've got it.

Chapter 6

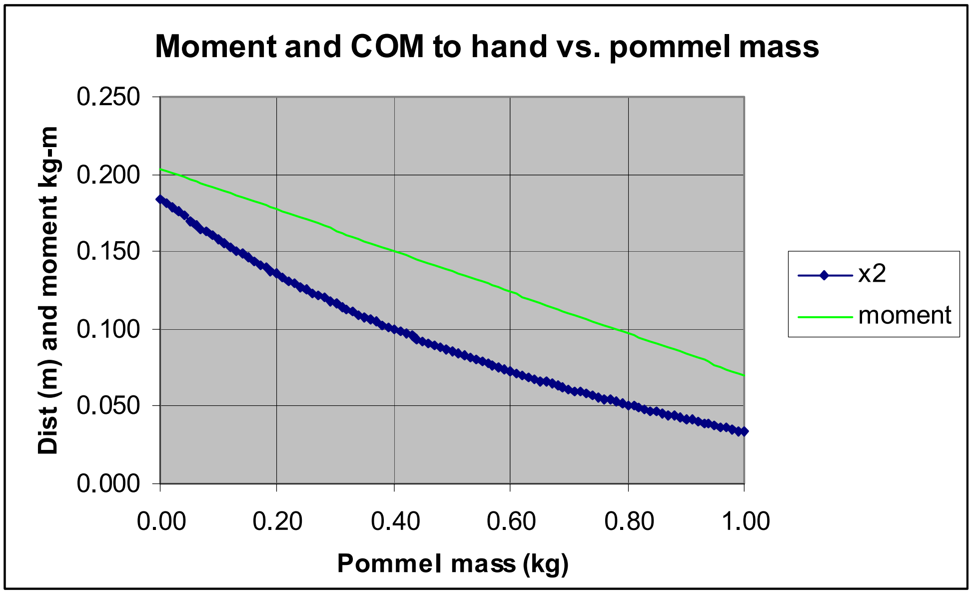
The Pommel

We all know about the pommel, or do we? By pommel I'm not talking about how it interfaces to your hand. That could be done much more easily with a piece of carved wood. Terminating the grip with some sort of knob can be done in many ways. Ways that don't increase the swords mass by a significant amount. But this large increase in mass is exactly what results from the medieval pommel. So, in terms of your grip, only the surface shape of the pommel matters, not its mass. The following analysis covers is the effect of this pommel mass on the sword as a whole. An important but generally ignored subject, that gets very interesting indeed.

Since $Q \cdot x$ is the ratio of inertia to mass, a way to spread these points toward the ends of the stick, thus making $Q \cdot x$ larger, would be to increase this inertia to mass ratio. The simplest way to do that is to move the mass from the middle of the stick to the ends. But that makes a barbell, which handles incredibly poorly. Looking at the equation $Q = \dfrac{I_{COM}}{m_{SWORD} \cdot x}$ brings up another method. Making x very small, by pulling the center of mass back close to your hand, must make Q get large. If Q gets large the percussion point shoots out to the far end of the weapon. Think of this trick as using a baseball bat gripped at the fat end, instead of the thin end.



The conventional view is that the pommel is used to adjust the balance point, which while of little importance by itself, does directly affect the static moment felt by the forearms when holding the weapon horizontally. But although the moment decreases linearly with increasing pommel weight, the balance point does not.



Let's go back to the modified Austrian saber, as I happen to have numbers on it. The chart shows how the value x, the distance from forefinger to center of mass, decreases in a non-linear fashion, and the moment decreases linearly. Eventually, both the distance x and the moment will go negative, as the center of mass passes under the forefinger. Needless to say this would make a poor weapon, both heavy and unable to deliver a descent blow. Given a pommel mass of 0.2 to 0.4 kg (7 to 14 ounces), which is probably where most weigh in, the decrease in moment is 87% to 73% of the original values, respectively. But this is bought at a price in weapon mass, which for the 7-ounce and 14-ounce pommels jumps to 118% to 136% of the original mass. At the right side of the graph, the weapon mass has nearly doubled.

By the way, the formula for calculating the new center of mass when weight is added is $\Delta COM = y \cdot \frac{\Delta m}{m + \Delta m}$, where y is the distance from the old center of mass to the added weight, and Δm is the added weight. The center of mass, of course, moves toward the added mass. This means the new x, the distance from the center of mass to your hand can be given by the formula $x_{NEW} = x_{OLD} - \Delta COM$, where ΔCOM is as defined above.

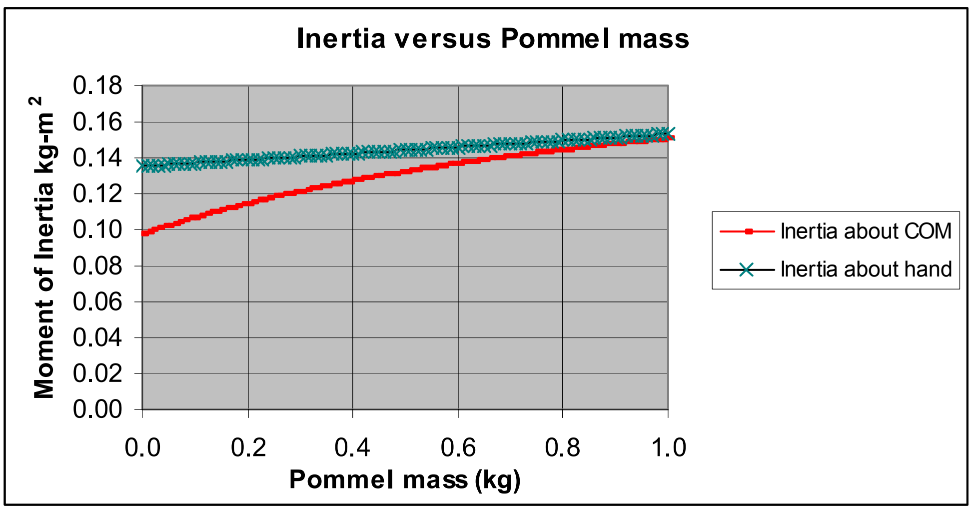
The Pommel's Effects on Moment of Inertia

Adding a pommel increases the weapons moment of inertia about its old center of mass, since adding any mass anywhere increases the $m \cdot x^2$ component. But it also pulls the center of mass back toward the added pommel, canceling some of this increase. You add the mass times it's distance to the old center of mass squared, but this added mass also gives you a new center of mass location. To get to the new center of mass location, subtract the new total mass times the distance, squared, that the center of mass moves. Mathematically, here's how you handle the moment of inertia calculations when adding any mass to a weapon.

Recalculating $\mathbf{I_{COM}}$ when adding mass to a weapon

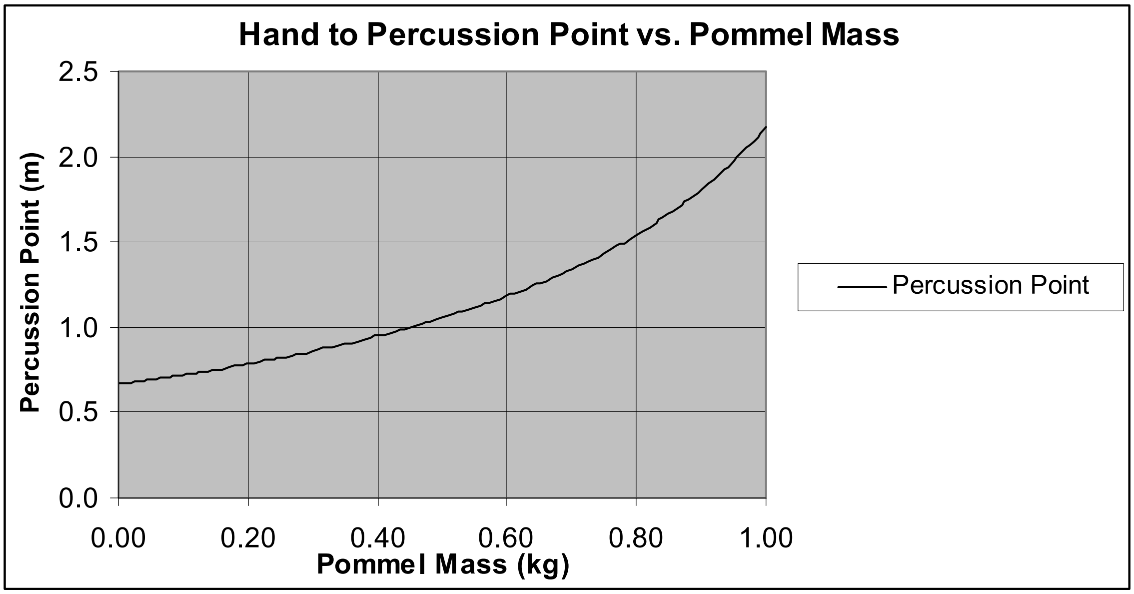
When adding a new mass, $\Delta\mathbf{m}$, to a weapon the mass changes as
$\mathbf{m_{NEW} = m_{OLD} + \Delta m}$
Given the distance **y**, from the old center of mass to the added mass, the center of mass of the weapon moves toward the added mass by a distance given as
$\mathbf{\Delta COM = y(\Delta m/m_{NEW})}$
The added mass directly increases the inertia about the old COM location.
$\mathbf{I_{COM\_MODIFIED} = I_{COM\_OLD} + \Delta m\, y^2}$
Using the parallel axis theorem we now shift the axis to the new COM location.
$\mathbf{I_{COM\_NEW} = I_{COM\_MODIFIED} - m_{NEW}\,(\Delta COM)^2}$
Or if you prefer to do everything in one step this changes to
$\mathbf{I_{COM\_NEW} = I_{COM\_OLD} + \Delta m\, y^2\,(1 - \Delta m/(\Delta m + m_{OLD}))}$

But what about the moment of inertia your hand feels? This moment of inertia is given by $I_X = I_{COM} + m_{SWORD} \cdot x^2$, where x is the distance from the center of mass to your hand. By adding a pommel x just got smaller, so even though I_{COM} increased by some small amount, the value you feel will increase much less.



This graph gives a good indication of the pommel's effect on the moment of inertia about the center of mass, and about the hand. The pommel has a negligible affect on the moment of inertia about the hand. For the 7-ounce pommel, it went up only 2.6%, and for the 14-ounce pommel, it went up only 5.3%.

Next we consider the effect of the pommel on percussion point location. The percussion point formula $Q = \dfrac{I_{COM}}{m_{SWORD} \cdot x}$ gives us one way to look at things. But as we adjust pommel mass, I_{COM} is varying a little, mass is increasing a great deal, and x is decreasing according to one of the above charts. An additional difficulty is encountered, in that the rearward movement of the center of mass must be subtracted from the increasing value of Q. An easier way to look at these changes would be to calculate $Q_X = \frac{I_X}{m_{SWORD} \cdot x}$, which is the percussion point distance calculated relative to the hand location x. As we've seen, the inertia about the hand increases very little with pommel mass, so I_X is more stable. The mass is increasing linearly, giving a decreased moment of inertia to mass ratio, which would normally pull the percussion point back, but x is decreasing even faster, pushing it out. The result of all this is shown in the following graph.



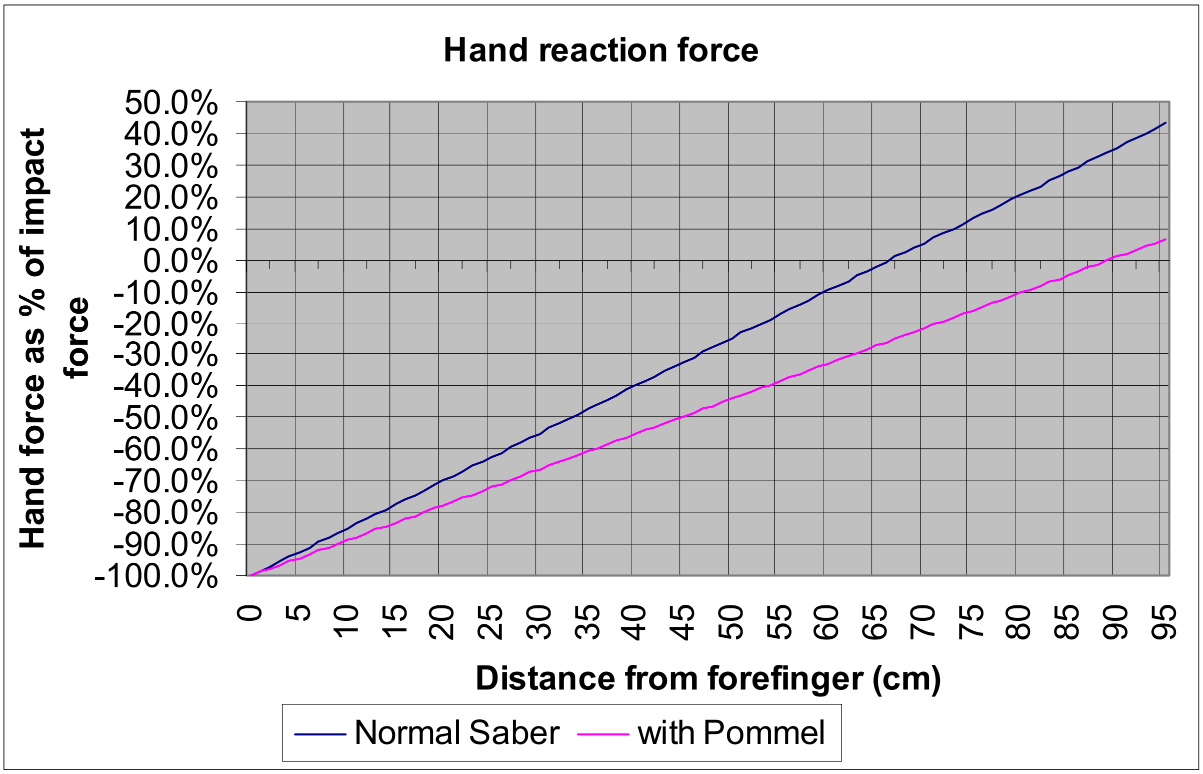
You can see that the percussion point is moving away, and at an ever-accelerating rate. As the pommel finally pulls the center of mass back to your hand, making x go to zero, the percussion point moves to infinity. This makes sense, because then you're holding the weapon by its center of mass, applying a single force with your hand merely causes it to move sideways, without any tendency to rotate. Keep in mind that the percussion point location is an exponential function. The distance from the center of mass to the percussion point, of any impact weapon, can vary from the square root of the moment of inertia to mass ratio, all the way to infinity.

Here is the percussion point range, stated mathematically. $\sqrt{\frac{I_{COM}}{m}} \leq Q \leq \infty$

The graph also shows that a pommel mass that's too high will also cause the percussion point to move past the tip of the sword, which is probably a very bad thing. This sabers actual length from forefinger to tip is 0.946 meters, and the percussion point distance exceeds this length with any pommel greater than 0.40 kg. This would probably be the maximum "balancing" the design would tolerate.

I modified a medieval Del Tin 2151 for better balance, because I'm picky. Unfortunately, at the time I didn't know all this percussion point stuff, and the result was a percussion point location 5.75 inches past the tip. The pommel was the same, so I don't know if the original percussion point was already past the tip or not. This points to the danger of balancing a sword, as opposed to adjusting its percussion point. As you can see from the graph, as x gets small, a change in x brings about a much larger change in Q. The more "balanced" the sword is, the more sensitive the percussion point is to small changes in x. To see how sensitive this is, look at the equation. $\frac{dQ}{dx} = \frac{-I_{COM}}{mx^2}$

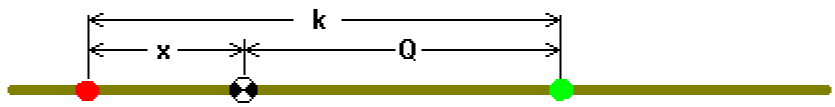
So as x gets small, providing balance, the percussion point is traveling toward the tip, at a rate inverse to the square of x. When you start with an unbalanced cavalry saber there's not much trouble, but as you approach a beautifully balanced medieval sword, bad things can happen with just small changes in pommel mass. So be very careful about adjusting a swords balance point. The percussion point is much more sensitive than the center of mass.



Just for completeness, here is the hand force verses impact force for the Austrian saber after adding a 341g pommel. Notice that for cuts near the tip the hand shock is much less than the unmodified saber.

Selecting a COM Position

If were designing an impact or cutting weapon of mass m and moment of inertia I_{COM}, that had to have some distance k between the hand and the hand's percussion point, what options do we have regarding balance point?



Given that $k = Q + x$, so $Q = k - x$, and $Q = \frac{I_{COM}}{m \cdot x}$, we have $kx - x^2 - \frac{Icom}{m} = 0$, which is a quadratic equation in x. The solution is given by $x = \frac{k \pm \sqrt{k^2 - \frac{4 \cdot I_{COM}}{m}}}{2}$, which can be rewritten as $x = \frac{k}{2} \pm \frac{\sqrt{k^2 - \frac{4 \cdot I_{COM}}{m}}}{2}$. Obviously, having $x = \frac{k}{2} \pm 0$ would represent the case of placing the center of mass directly in between the hand and the percussion point. This, for a given moment of inertia to mass ratio, also sets up the minimal distance between the hand and the hand's percussion point. As we try to increase the distance k, to give our weapon longer reach while leaving the moment of inertia to mass ratio alone, we automatically get two solutions for x. Each solution represents a pair of center of mass locations, which are equally distant from the center point, located between the hand and the hand's percussion point. The solution to $x = \frac{k}{2} - \frac{\sqrt{k^2 - \frac{4 \cdot I_{COM}}{m}}}{2}$ moves the center of mass nearer your hand, while the solution to $x = \frac{k}{2} + \frac{\sqrt{k^2 - \frac{4 \cdot I_{COM}}{m}}}{2}$ moves the center of mass further from your hand. The pair of solutions dictates that you hold the baseball bat by the thin end, or the thick end. Either way, you've got a percussion point in your hand, and its mate is further out. Note that the distance between the two percussion points, for a given center of mass location and moment of inertia to mass ratio, is a constant, no matter which way you hold the bat. But the distance from your hand to the center of mass is quite different for the two cases.

Also note that there is a minimum value of k below which $k^2 - \frac{4 \cdot I_{COM}}{m}$ becomes negative, which leaves the equation for x with no solutions. This means that there can't be any center of mass location that will generate percussion points of the desired distance apart, or k. So solutions to this problem must have $k^2 \geq \frac{4 \cdot I_{COM}}{m}$, or $(Q + x)^2 \geq \frac{4 \cdot I_{COM}}{m}$. Considering that the shortest distance for k also corresponds to the case where $Q = x$, the inequality becomes $(x + x)^2 \geq \frac{4 \cdot I_{COM}}{m}$, $4x^2 \geq \frac{4 \cdot I_{COM}}{m}$ or just $x \geq \sqrt{\frac{I_{COM}}{m}}$.

Comparing a Pommel to a Tang Extension

The question naturally arises, why not just extend the tang, making a longer handle, and not use a pommel at all. This is the basic design approach of a katana, so let's compare the two. With some given existing blade, we want to have a given percussion point to hand distance k. We can examine the end results of either adding mass to the pommel, or extending the tang.

With the pommel, all the mass is being concentrated near the hand, adding very little to the moment of inertia about the hand. The extended tang has mass being added far past where the pommel would be, and this mass adds more to the moment of inertia of the hand than does the pommel. However, this mass being added further out does more to move the center of gravity back, so less total mass is needed in order to achieve the same objective. However, if large adjustments to the percussion point are needed, the tang may end up being unreasonable long.

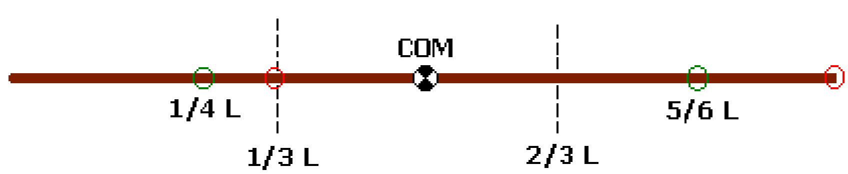
Here are the results of modifying a basic blade with the two different methods.

We start out with a 1.0 kg blade, with I_{COM} = 0.08 kg-m^2. The original COM location is 8 inches, and the handle length is 6 inches. This gives an initial percussion point to forefinger distance of 23.5 inches. Suppose we want to increase this to 36 inches.

With a pommel:
Extra mass is 0.397 kg, bringing the weapons mass to 139.7% of the original.
Moment of inertia about the center of mass is 144.92% of the original.
Moment of inertia about the forefinger is only 107.6% of the original.
Moment of inertia about the center of the handle is only 101.46% of the original.
The new balance point is 4.02 inches from the forefinger.

With an extended tang, of dimensions 1.0 inches by 1/4 inch:
Extra mass is 0.229 kg, bringing the weapons mass to only 122.9% of the original.
Moment of inertia about the center of mass is 147.04% of the original.
Moment of inertia about the forefinger is 111.61% of the original.
Moment of inertia about the handle is 104.40% of the original.
The new balance point is 4.73 inches from the forefinger.
The added tang extension is 7.12 inches long, making the total handle 13.12 inches.

So the weapons with the tang extension is a little lighter, but very slightly less maneuverable. It balances a bit further out, but not by much. The main difference is the much longer handle. In fact, you can't make the percussion point distance very large without also having a long handle. You can use a katana with one hand, but you can't make a single handed katana. To maintain the percussion point location of a two handed katana, it would still need to retain the long handle, or have a pommel, in which case it certainly wouldn't be a katana. If you merely shorten the handle, it would become a short cavalry saber.



Recall the stick, and remember your training. If you hold the stick by the end then the corresponding percussion point is 1/3 back from the tip, as show by the red circles. To move the percussion point further toward the tip, you have to move your hand up. In effect, you have to make the handle longer and the blade a bit shorter. For a fixed blade length, you can make the handle longer, which leaves you with a longer stick, which you're holding further from the end of the handle. Instead of giving yourself an uncomfortably long handle, which can bind up your own freedom of motion, you can concentrate that handle into a dense ball, or pommel. If you taper the mass of the stick, thinning the ends, to make it more maneuverable, the ratio of inertia to mass drops, the percussion point moves back toward your hand, and you need an even longer handle, or a heavier pommel. If you don't use a pommel, the handle length limits your corresponding blade length, which must remain in proportion to the blade length. Otherwise you must sacrifice having a percussion point that's very close to the tip.

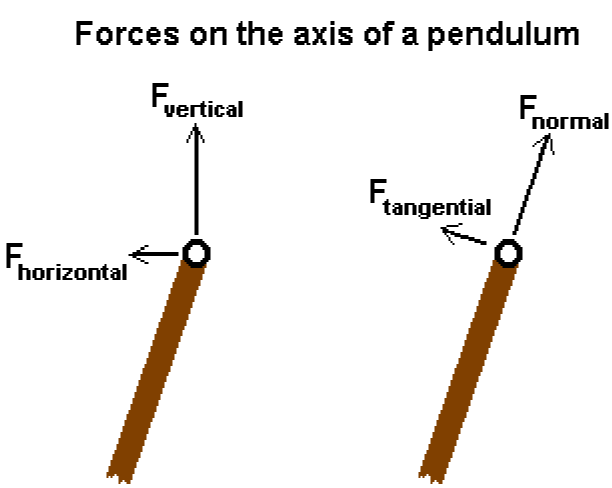
A Pendulum's Percussion Point

As stated in an earlier chapter, a pendulum's period of oscillation is determined by the distance from its axis of rotation to the percussion point, relative to that axis of rotation. Before going further, I'll explain why this is, since I couldn't find it discussed in detail in any dynamics books. Most textbooks give only a short treatment of pendulums, and don't mention the percussion point of a complex pendulum at all, prefering to treat the pendulum as a point mass suspended by a massless string. Even the few textbooks that discuss a compound pendulum just rely on the formula for the period, which includes the pendulums moment of inertia. But they don't explain *why* a pendulum would swing according to the percussion point, other than as a byproduct of the formula for a pendulum's period. The fundamental reason occurred to me on a long drive from Texas, so here it is.

When you swing a pendulum, the only external forces on it are gravity, which acts equally on every bit of the pendulums constituent masses, and the forces applied at the pivot point, or axis of suspension. Obviously there is an upward, vertical component of force on the axis, to counteract the force of gravity on the pendulum. The vertical force varies throughout the swing, due in part to centrifugal forces exerted by the pendulum, and in part to the fact that at the apex of the swing the mass of the pendulum is partly in free-fall. You felt these forces in childhood, riding on a swing set. You feel light at the top, and heavy as you speed through the bottom of the swing. So the vertical component of force varies throughout the pendulum's swing,

Likewise, there must exist horizontal forces at the axis, otherwise the pendulum's center of mass would remain in a fixed horizontal position, since in the absense of applied forces the pendulum's center of mass could experience no acceleration, and thus no change in horizontal velocity. If you want to feel this force, simply grab up a heavy object and swing it like a pendulum. You'll notice that you have to apply side-to-side forces to keep your hand still. Otherwise it shifts from side to side. Centrifugal force is also contributing to this horizontal component, and gets stronger as the amplitutde of the swing increases.

We can easily change our reference system to break the horizontal and vertical forces, applied at the axis, into normal and tangential forces applied to the pendulum. The normal force acts in direct line along the axis of the pendulum, and merely changes the tension along the pendulum's shaft. The normal force will vary throughout the swing, and corresponds pretty exactly to the forces you felt in your childhood swingset riding days. The tangential force is always, by definition, applied perpendicular to the rod, and also changes throughout the swing.



Interestingly, the normal force can't affect the swing of the pendulum. Since it's always applied in direct line with the pendulum's center of mass, it can't affect the rotation rate of the pendulum. Basically, it can't apply any torque, because it has no moment arm relative to the center of mass. No torque means there's no angular acceleration, and no angular acceleration means there's no change in angular velocity. This is why we can build stable rockets that have the engine pushing from the back. In some of Robert Goddard's first experiments, he put the engine on top, since common sense will tell you that when you pull an object, it will dutifully follow along, but when you push it, it will veer to one side or the other, and careen off in some random direction. Common sense was wrong, and Goddard moved his engine to the back of the rocket. As long as the push remains directly in line with the objects center of mass, it can't influence the object's course. The error in reasoning comes from thinking that the engine pushes vertically, even as the rocket starts to lean over. But the engine's thrust always remains aligned with the rocket, not in some fixed direction, so our air-to-air missiles can be pushed from the back, and our pendulum's swing is unaffected by the normal force on the axis of rotation.

Gravity also doesn't influence the swing, which may come as a bit of a shock, since what else is there? If the fixed axis suddenly broke, the pendulum would fall with its center of mass accelerating downward, the pendulum itself rotating around the center of gravity at whatever constant angular velocity was present when the axis broke. As Einstein showed, you can get rid of gravity in an analysis by assuming that falling objects are not subject to any external forces; they are just in free fall. Objects that aren't falling are subject to external forces that are causing them to resist free fall. Many problems are greatly simplified by this change in reference frame. Essentially, he said that a person in a room couldn't tell if he's subject to gravity at the earth's surface, or if he's in a rocket ship that is accelerating at 1G. We could likewise say that there are no external forces on the pendulum except through its attachment to the axis, and the axis is attached to the side of a spaceship that's accelerating at 1G.

So we have no external forces on the pendulum other than through the axis, and the normal force along the pendulum's shaft can't affect the rotation. That only leaves the tangential forces at the axis to affect the swing. As we've already established, tangential forces applied to some point of an object cause the object to pivot around the corresponding percussion point, which feels no torque. So the pendulum's motion is merely an odd version of our waggle test, and we're part way to a solution.

Note that the percussion point undergoes no torque due to the tangential forces at the axis. Likewise, whenever we make a simple pendulum by suspending a ball from a string, the ball also undergoes no torque, since strings can't transmit torque, only tension forces, which are of course colinear with the string. So if we suspend the ball so that it matches up with our percussion point, we have two points that are equidistant from the axis of rotation, both of which undergo no torque, only forces directed straight toward the axis of rotation. We know that the ball suspended from the string will swing according to the equations of a simple pendulum.

Both the ball and any object located at the pendulum's percussion point are subject to the same forces. Neither feels any torque, so their tangential accelerations, relative to a line directed at the pendulum's axis of rotation, must be zero. Both also experience the same accelerations directed toward the axis of rotation, since their distance from the axis is fixed, in one case by the string, in the other by the shaft of the pendulum. If the distance can't change, there is no relative velocity. This means the relative velocity itself undergoes no change, which means no difference in the accelerations toward or away from the axis of rotation. Two masses subject to the same accelerations in both planes of motion must react the same, as Galileo's experiments at the Leaning Tower of Piza demonstrated, even if he really didn't perform them. This means that the percussion point of the real pendulum swings with the same rate as a simple pendulum of similar length.

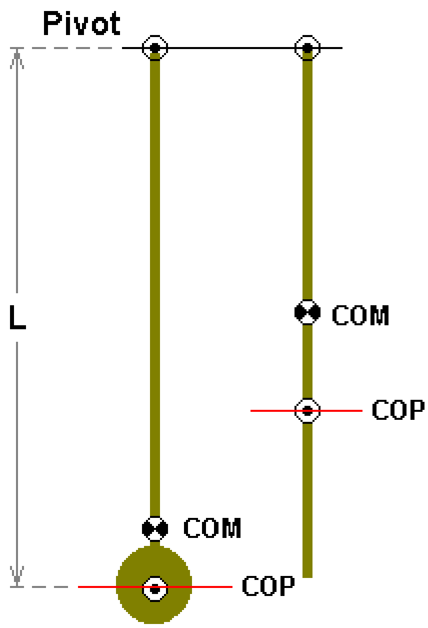
Instead of suspending the ball from a string, you could also add a little horizontal shelf to the real pendulum, so we can lay the ball on the shelf while having the ball's center of mass adjacent to the real pendulum's percussion point. During the pendulum's swing, the ball will happily ride on the shelf, seemingly oblivious to all the swinging. Roughly stated, the ball is at rest relative to the shelf.

Let's shift reference frames and imagine a giant pendulum attached to the side of an accelerating space ship. You're sitting in a cabin at the pendulum's percussion point. Your legs tell you that the local acceleration is alternately increasing and decreasing, just as it does on a swingset. Your inner ear tells you that you're also undergoing slight rotations throughout the swing. Looking up, you see the shaft of the pendulum stretching above you, apparently vertical and straight up. The ship seems to be changing its course, first pointing to the right of vertical, then to the left. But if you place a ball at your feet, it just sits there, but rolls a few degrees right and left as the pendulum swings. From your standpoint, you are at rest, and the ship is pulling the axis of the pendulum from side to side. You know you must be at the percussion point relative to the axis, because if you were closer to the axis, or further away, you'd be getting knocked into the sides of the cabin with each swing. If the ship stops accelerating, you enter free fall, and float about the cabin. Now if the ship moves perpendicular to its previous course, back and forth, it's doing a waggle test. Again you remain at rest, floating in the cabin, and watch as the room rotates slightly through each waggle. If you look at a fixed star, the shaft of the pendulum will move to the left and right. If you were far from the percussion point, the cabin's walls would again be slamming into you. From the perspective of your cabin at the pendulum's center of percussion, the simple waggle test and the pendulum test look the same.

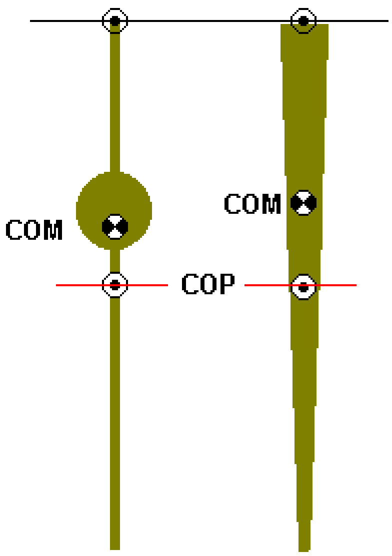
So in short, the waggle test and the pendulum test are using similar methods to reveal identical dynamic properties.

Adding a Pommel to a Pendulum

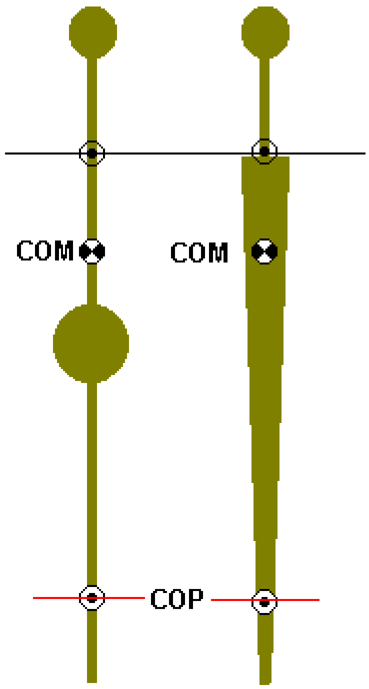
As stated earlier, most textbooks address a simple pendulum, which doesn't bring up percussion points. But once you start studying complex pendulums, lots more interesting physics shows up. We know how a simple pendulum swings, and a heavy weight connected to a rod is a pretty close analog. But lets start trying out different shapes for a pendulum, and see what happens. Given a standard looking pendulum, let's remove the big weight, then and just leave a straight rod, which as we've already determined has a percussion point, relative to an end, located 2/3 of it's length from that end. So the straight rod will swing much faster that the rod with a large mass at the end. A quick illustration will show what we're doing.



But as we've already observed, a tapered rod has a percussion point, relative to the heavy end, that's much less than a uniform rod. So our pendulum ends up with a much faster period, and a much shorter axis to percussion point distance, as shown in the following illustration. The question becomes how to keep from adding mass way down, which would be hard to swing, while still slowing down the frequency of the pendulum.



To accomplish this amazing feat, we merely need to add mass where it increases the moment of inertia of the pendulum, while not feeding the energy in the pendulum's swing. If we add mass above the axis of rotation, that mass adds inertia just as it would if we added it an equal distance from the axis, but below the axis. But above the axis the mass doesn't feed the energy the pendulum can extract from gravity, it detracts from it. Mass below the axis gains potential energy, according to $PE = mgh$, as the pendulum moves away from vertical. But above the axis, mass loses potential energy just as mass below the axis gains it. So mass above the axis cancels some of the potential energy available below the axis, yet still adds to the moment of inertia. It helps in both ways, and that is one amazing demonstration of the pommels effect. Grab up some handy piece of rod, and set it swinging like a pendulum, with it held by your fingers as if you were holding the cross guard of a sword. Then clamp some vice grips, or any handy mass that will stay on top, to the upper end of the rod, where a pommel would be. The frequency of the swing drops dramatically, as if you had a very long pendulum.



The underlying reason comes from the physics of pendulums. One form of the equation for the resonant frequency of a pendulum is $\omega = \sqrt{\frac{m \cdot g \cdot L}{I_{PIVOT}}}$, where m is the mass of the entire pendulum apparatus, g is the acceleration of gravity, L is the distance from the axis of rotation to the pendulum's center of mass location, and I_{PIVOT} is the moment of inertia about the axis of rotation, or pivot point. The frequency, in cycles per second, is just $2 \cdot \pi$ times the frequency given in radians/second, so $f = 2 \cdot \pi \cdot \sqrt{\frac{m \cdot g \cdot L}{I_{PIVOT}}}$. The pommel is increasing the moment of inertia, increasing the length, yet decreasing the distance from the axis of rotation to the center of mass. Any mass added above the axis of rotation must decrease the natural frequency of the pendulum, and thus increase the percussion point distance from the axis of rotation. This is also a handy way to measure the moment of inertia of an object. Given that $\omega = 2 \cdot \pi \cdot f$, we have $2 \cdot \pi \cdot f = \sqrt{\frac{m \cdot g \cdot L}{I_{PIVOT}}}$, the period is given as $\frac{2 \cdot \pi}{\tau} = \sqrt{\frac{m \cdot g \cdot L}{I_{PIVOT}}}$. This gives us $\frac{4 \cdot \pi^2}{\tau^2} = \frac{m \cdot g \cdot L}{I_{PIVOT}}$, which is easily solved for I_{PIVOT}.

$$I_{PIVOT} = \frac{m \cdot g \cdot L \cdot \tau^2}{4 \cdot \pi^2}$$

This is the equation for the moment of inertia of a pendulum, about the axis of rotation.

Using this equation, we can suspend the sword by its hilt, time its swings to find the period of oscillation, measure the distance from the point of suspension to the center of mass, and then calculate the moment of inertia about the axis to a very good degree of precision. Using this moment of inertia, we can use the parallel axis theorem to find the moment of inertia about the sword's center of mass. This is given as just

$I_{COM} = I_{PIVOT} - m \cdot L^2$, and in another form we get

$I_{COM} = \dfrac{m \cdot g \cdot L \cdot \tau^2}{4 \cdot \pi^2} - m \cdot L^2$. **This is the equation for the moment of inertia of a pendulum, about the pendulums center of mass.**

Chapter 7
Calculating Required Pommel Mass

There is a handy mathematical shortcut to sizing a pommel, which also shows some really interesting physics. If you try to solve some of the previous equations for required pommel mass, you just end up in a hairball. The easiest way to use them is to build a spreadsheet in which you.

A) Plug in the mass, length, moment of inertia, and handle length.
B) Plug in a guess for the required mass of the pommel.
C) Calculate the resulting change in position of the center of mass.
D) Calculate the resulting moment of inertia about the new center of mass.
E) Calculate the resulting percussion point location, relative to the forefinger.

Unfortunately, this process is iterative, tedious, and makes automatic solutions during sword optimizations difficult in a spreadsheet. So let's look at some other ways to approach the problem.

Given in this problem are the parameters of the blade. We already assume we know its mass, moment of inertia, and balance point. We are going to be adding a new mass to the weapon, which means the finished weapon will have parameters that are quite different from those of the bare blade. So we're trying to solve for a desired percussion point location, relative to the hand.

One method would be to use our equation for required center of mass location,

$$x = \frac{k}{2} \pm \frac{\sqrt{k^2 - \frac{4 \cdot I_{COM}}{m}}}{2}$$

. But we also have another little cheat for finding the center of mass location, x. The center of mass of an object can be found by dividing its static moment, which we've been using as $m \cdot x$, by the object's mass m. If you treat the blade as a series of individual masses, all strung together, you can find the resulting balance point by summing together the product of each mass, multiplied by its distance from some fixed point, then dividing this sum by the total mass. The reference point is called the datum, and the result of the calculation is the distance from the center of mass to the datum. This is a handy trick that all aircraft pilots have to perform, to make sure that the aircraft's balance point is sufficiently close to, and forward of, the wing's aerodynamic center.

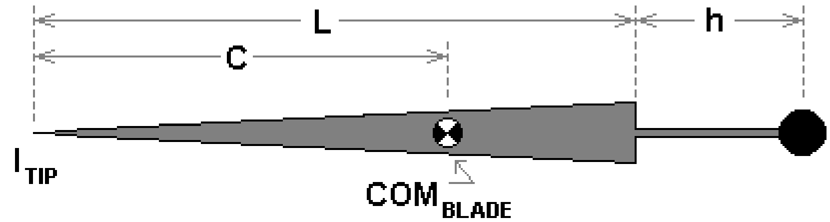
$$COM = \frac{\sum_{i=1}^{n} m_i \cdot x_i}{\sum_{i=1}^{n} m_i} \qquad COM = \frac{\sum_{i=1}^{n} m_i \cdot x_i}{m_{TOTAL}}$$

We already know the mass and center of mass location of the blade, which are easily measured. We also know exactly where we want to place the pommel. So if we know the mass of the pommel, the resulting center of mass location, relative to any point, is calculated quite easily.

$$COM_{NEW} = \frac{m_{BLADE} \cdot COM_{OLD} + m_{POMMEL} \cdot LOC_{POMMEL}}{m_{BLADE} + m_{POMMEL}}$$

. This new location for the center of mass is given relative to the same point that the old location was, which we call the datum. The location of the pommel is also given as relative to the datum. All this serves to give us another solution for x, which we can plug into the equation for the required center of mass location that gives a desired percussion point distance. Unfortunately, if done relative to the hand we end up in another big hairball.

So we'll flip things around and try to solve the equations as if we're designing a club, putting the pommel on the far end, to extend the percussion point out some desired distance.



Here we have a bare blade of known mass, balance point, and moment of inertia about the center of mass. We also know the length of the handle, which is given as h. We want to figure out the pommel mass that will move the percussion point, relative to the forefinger, to the tip. Since percussion points always come in pairs, this is the same as calculating the tip's percussion point, and making sure that it is past the tip by the exact distance L. In this method, I'm calculating to place a percussion point relative to the tip, but we could just as easily lie about the tip location. The math comes out the same. From wherever you want your hand's percussion point to end up, let L be the distance from that point to your hand, and C be the distance from that point to the bare blade's center of mass.

We can easily calculate the bare blade's inertia about the tip, by using the parallel axis theorem, so $I_{TIP\_BLADE} = I_{COM\_BLADE} + m_{BLADE} \cdot C^2$. We can also calculate the final sword's inertia about the tip, since we are just adding a pommel of mass m_{POMMEL} at a distance of $L + h$. By the definition of moment of inertia, we have $I_{TIP\_SWORD} = I_{TIP\_BLADE} + m_{POMMEL} \cdot (L+h)^2$, or $I_{TIP\_SWORD} = I_{COM\_BLADE} + m_{BLADE} \cdot C^2 + m_{POMMEL} \cdot (L+h)^2$.

Using our previously derived equation for the location of the center of mass, we get $COM_{SWORD} = \dfrac{m_{BLADE} \cdot C + m_{POMMEL} \cdot (L+h)}{m_{BLADE} + m_{POMMEL}}$. This is the distance from the tip to the final sword's center of mass. Using our equation for the total point distance, often expressed as $Q + x = \dfrac{I_X}{m \cdot x}$, where x is the distance from one percussion point to the center of mass, I_X is the moment of inertia around that point, and $Q + x$ is the distance in between the pair of percussion points. In this case, we want $Q + x = L$. Our equation for the percussion point distance becomes to $L = \dfrac{I_{TIP\_SWORD}}{(m_{BLADE} + m_{POMMEL}) \cdot COM_{SWORD}}$, where COM_{SWORD} is the distance from the tip to the final location of the sword's center of mass. But looking at the equation for COM_{SWORD}, we see that it is given as the static moment divided by the total mass, and in the denominator of the percussion point equation we turn around and multiply this by the total mass. The masses cancel, leaving us with just the static moment, which we had to calculate anyway, to figure out the new location for the center of mass. This simplifies things to $L = \dfrac{I_{TIP\_SWORD}}{m_{BLADE} \cdot C + m_{POMMEL} \cdot (L+h)}$. Substituting in for I_{TIP_SWORD} we get

$$L = \frac{I_{TIP\_BLADE} + m_{POMMEL} \cdot (L+h)^2}{m_{BLADE} \cdot C + m_{POMMEL} \cdot (L+h)}$$. This becomes

$$m_{POMMEL} \cdot \left(\frac{h^2}{L} + h\right) = m_{BLADE} \cdot C - \frac{I_{TIP\_BLADE}}{L}$$, which is easily solved as

$$m_{POMMEL} = \frac{m_{BLADE} \cdot C - \frac{I_{TIP\_BLADE}}{L}}{\frac{h^2}{L} + h}$$. This can also be cleaned up to give

$$m_{POMMEL} = \frac{m_{BLADE} \cdot C \cdot L - I_{TIP\_BLADE}}{h \cdot (h+L)}$$. **Equation for required pommel mass**

In this equation, since the blade's moment of inertia about the tip is generally a linear function of blade mass, the numerator in the pommel mass equation, $m_{BLADE} \cdot C \cdot L - I_{TIP\_BLADE}$, tends to be linear with blade mass. In general, the pommel mass must increase linearly with blade mass. We also knew this intuitively, since two identical swords, with identical percussion points, could be tack welded together, which would make a blade with exactly same percussion point, but also with exactly twice the blade mass, and twice the pommel mass. It doesn't much matter if you worry about the mass of the cross guard, since it's so close to the forefinger that it has a negligible affect on the percussion point, relative to the forefinger. The key to sizing a pommel is measuring the bare blade, which of course has a tang, and possibly the handle.

Another thing that stands out in the equation is its extreme sensitivity to handle length. If you pick a short handle, the denominator in the equation becomes very small, and the pommel mass skyrockets. When you really try to cramp up the grip, as on some single-handed Viking and medieval swords, you do end up with a really big pommel. If you let the grip get longer, the pommel gets much smaller, so on a giant two-handed sword, the pommel is very roughly the same size as is found on a rapier. The rapier, with its short little handle, has a surprisingly large pommel. The short handle to blade length ratio is the big factor driving this. If you let the handle get a bit longer, you have a katana, and don't need the pommel at all. If you set the pommel equation to zero, you can back calculate the length of true, exposed blade, based on the bare blade you started with.

The Effect of Blade Mass Taper on Pommel Mass

I was running through some numbers, to determine the difference in required pommel size for a variety of different mass taper ratios. I wanted the forefinger's percussion point to remain at the very tip of the sword. Basically, as the sword moves from a uniform rod, with no mass taper, to an acutely pointed one, and all phases in between, I needed to know how the pommel size changes to set the percussion point. The results were startling, which is what drove me to actually come up with the equation for required pommel mass.

Given a blade with no mass taper, which is essential a rod or piece of flat stock, it of course balances in the middle, and the moment of inertia is given by $I_{TIP\_BLADE} = \frac{m \cdot L^2}{3}$. Looking back at the equation for pommel mass, which was $m_{POMMEL} = \frac{m_{BLADE} \cdot C \cdot L - I_{TIP\_BLADE}}{h \cdot (h+L)}$, and substituting in what we know about the rod, we get $m_{POMMEL} = \frac{m_{BLADE} \cdot \frac{L^2}{2} - m_{BLADE} \cdot \frac{L^2}{3}}{h \cdot (h+L)}$.

This simply reduces to $m_{POMMEL} = \frac{m_{BLADE} \cdot \frac{L^2}{6}}{h \cdot (h+L)}$. If we provide complete mass taper, where the blade mass comes linearly to zero at the tip, the balance point, relative to the tip, is 2/3 the length, so $C = \frac{2}{3} \cdot L$. As previously derived, the moment of inertia about the tip will be $I_{TIP\_BLADE} = \frac{m \cdot L^2}{2}$. So for absolute linear mass taper we get $m_{POMMEL} = \frac{m_{BLADE} \cdot \frac{2 \cdot L^2}{3} - m_{BLADE} \cdot \frac{L^2}{2}}{h \cdot (h+L)}$. This also simplifies to $m_{POMMEL} = \frac{m_{BLADE} \cdot \frac{L^2}{6}}{h \cdot (h+L)}$. So the required mass of the pommel is the same for a rod, and for absolute and linear blade taper. For states in between, we need merely note that each of those blades can be modeled as a combination of a blade of constant mass with a blade of linearly decreasing mass. Both blades have the same percussion point, and if tack welded together, the resulting blade also has the same percussion point. Punching this into a simple spreadsheet will show that all blades that have a linear decrease in mass require the same pommel, if their mass and length is the same.

If the mass is allowed to vary in a non-linear fashion, the pommel still doesn't vary by much. If you make a blade whose mass varies with the square root of the length, and another blade that varies with the square of the length, which will feel very different, the required pommel mass only varies by about 11%! Required pommel mass is completely unaffected by linear mass taper of the blade, and only very slightly affected by any type of mass tapering. It is relatively linear with blade mass, the square of the blade length, and roughly inverse to the square of hilt length, relative to blade length. However, if you linearly taper the last 1/3 of the blade, the blade as a whole has a very non-linear taper, and you'll need to up the pommel mass by about 9%.

So a reasonable starting point, to guess pommel mass, would be to assume some sort of linear blade taper, and use the simplified equation, which is absolutely correct for linear blade taper, and a pretty close estimate for other types of blades. This equation doesn't require any knowledge of the moment of inertia of the blade, which is a big plus. Given that the mass of a sword is mostly the mass of the blade and the mass of the pommel, and the mass of the cross-guard has negligible affect on percussion point location, and thus pommel mass, we can use this equation to back calculate a reasonable estimate of pommel mass on authentic swords. However, this calculation requires us to first subtract the mass of the cross-guard from the mass of the sword, so other methods might be more accurate.

$$m_{POMMEL} = \frac{m_{BLADE} \cdot L^2}{6h \cdot (h + L)}$$

Equation for close (within 10%) estimate of required pommel mass

This is interesting stuff, indeed. I wouldn't have ever expected the pommel to be so insensitive to the blade's distal taper, which has such a tremendous affect on feel and performance. Yet this is what I saw in my spreadsheets, which made me think something really deep was going on. As cosmologists say, if a number is very, very near zero, it probably is zero. Such was the case with my percussion point spreadsheet. As in illustration of what this kind of data looks like, here is the little spreadsheet that told the tale.

	METRIC				***ENGLISH***			
	full taper	1/10 taper	no taper		full taper	1/10 taper	no taper	
blade mass	0.750	0.750	0.750	kg	1.65	1.65	1.65	lbs
blade length	1.000	1.000	1.000	m	39.37	39.37	39.37	inches
COM location	0.333	0.400	0.500	m	13.12	15.75	19.69	inches
I_{COM}	0.042	0.055	0.063	kg-m^2	0.987	1.302	1.480	lb-ft^2
COM to tip	0.667	0.600	0.500	m	26.25	23.62	19.69	inches
handle length	0.203	0.203	0.203	m	8.00	8.00	8.00	inches
pommel mass	0.511	0.511	0.511	kg	1.12	1.12	1.12	lbs
ΔCOM	0.217	0.245	0.285	m	8.56	9.63	11.22	inches
new COM	0.116	0.155	0.215	m	4.56	6.12	8.46	inches
new I_{COM}	0.129	0.166	0.213	kg-m^2	3.059	3.922	5.040	lb-ft^2
new mass	1.261	1.261	1.261	kg	2.77	2.77	2.77	lbs
I_{FINGER}	0.146	0.196	0.271	kg-m^2	3.460	4.644	6.420	lb-ft^2
COP to COM	0.884	0.845	0.785	m	34.81	33.25	30.91	inches
tip to COP	0.000	0.000	0.000	m	0.00	0.00	0.00	inches
static moment	0.146	0.196	0.271	kg-m	12.655	16.986	23.482	inch-lbs

Note that the resulting blades have identical pommel masses, and identical tip to percussion point distances. If you investigate further, these weapons all show a perfect linear relationship between the moment of inertia about the finger, listed as I_{FINGER}, and the static moment, listed on the bottom line of the spreadsheet. Once we quit balancing the blades for feel, and start making percussion point distance constant, the moment of inertia about the finger is linearly related to the static moment. You can truly feel, in a static test, the dynamic "feel" of these swords. This has important implications, but I'll save that for another section.

Also note that these swords have widely varying balance points, static moments, and moment of inertias. In the hands of a 20th century replica maker, the blade with a high static moment would get a much larger pommel, to make it "feel" historically accurate. This would obviously drive the percussion point way off the end of the tip, and make it swing poorly. It would also mean that all impacts would be way inside the percussion point, so it would always kick back into the wrist.

The lighter sword, with full taper, would likely get a smaller pommel, making it act more like a cavalry saber.
Another important point is that you won't find any simple linear relation amongst them that correlates sword weight vs. static moment, or sword weight versus balance point. Authentic museum pieces show the same perplexing balance point and static moment data. Attempts to find a simple linear relation failed, or we would all be using it to construct better replicas. But I doubt a serious attempt was made to uncover the true meaning of percussion point, so people just assumed that balance point was mostly a matter of taste. With 19th and 20th century researchers tending to regard the old weapons as ancient, crude implements, suitable only for hacking away at an opponent, as opposed to refined fencing arts, no wonder we've been left thinking that balance is a matter of preference and individual style.

	METRIC				***ENGLISH***			
	full taper	1/10 taper	no taper		full taper	1/10 taper	no taper	
blade mass	0.750	0.750	0.750	kg	1.65	1.65	1.65	lbs
blade length	1.000	1.000	1.000	m	39.37	39.37	39.37	inches
COM location	0.333	0.400	0.500	m	13.12	15.75	19.69	inches
I_{COM}	0.042	0.055	0.063	kg-m^2	0.987	1.302	1.480	lb-ft^2
COM to tip	0.667	0.600	0.500	m	26.25	23.62	19.69	inches
handle length	0.203	0.203	0.203	m	8.00	8.00	8.00	inches
pommel mass	0.265	0.511	0.880	kg	0.58	1.12	1.94	lbs
ΔCOM	0.140	0.245	0.380	m	5.52	9.63	14.95	inches
new COM	0.193	0.155	0.120	m	7.61	6.12	4.74	inches
new I_{COM}	0.098	0.166	0.263	kg-m^2	2.322	3.922	6.222	lb-ft^2
new mass	1.015	1.261	1.630	kg	2.23	2.77	3.59	lbs
I_{FINGER}	0.136	0.196	0.286	kg-m^2	3.219	4.644	6.781	lb-ft^2
COP to COM	0.500	0.845	1.340	m	19.69	33.25	52.75	inches
tip to COP	0.307	0.000	-0.460	m	12.08	0.00	-18.12	inches
static moment	0.196	0.196	0.196	kg-m	16.986	16.986	16.986	inch-lbs

Here are the same three blades, but with pommels that were chosen to match the static moment that the 1/10 taper model had previously shown. In essence, I've just chosen pommels to match the static "feel" of the weapons. Note that the fully tapered model, which had previously had the shortest COM location, now has the highest, balancing 7.6 inches from the forefinger. It had been only 4.5 inches back. But we've also now made it a lightweight sword, whereas before all the swords the same weight. It doesn't maneuver much better, having a moment of inertia about the finger that's still 96% of its previous value. The moment of inertia calculated about the center of the handle, which I didn't bother to show, is a full 99% of its previous value. If was a very maneuverable sword with the heavy pommel, and it's not significantly improved by the light pommel. It just "feels" a bit beefier. But look what happened to the percussion point! It's now a full foot short of the tip. Strikes with the tip will now cause the handle to kick into your fingers. But if you've been incorrectly conditioned by the saber community to cut 1/3 back from the tip, this sword will cut smoothly, with very little hand shock.

The sword with the tip-heavy blade still lacks maneuverability. But hasn't been made much worse. The inertia about the forefinger has gone up 5%, and about the center of the handle it's gone up only 4%. The mass has gone up 73%, but after all, this is a medieval sword, and people think those are supposed to be really heavy! But it's balanced well, with the center of mass only 4.7 inches from the forefinger, which is much better than the previous 8.5 inches.

Unfortunately, though, its percussion point has shot a foot and a half past the tip of the blade. But this is the kind of thing you expect in weapons that are balanced for "feel". It's also representative of the kind of reproductions we get, even from very reputable manufacturers. I've actually measured worse percussion point locations, from top-flight smiths. I've also encountered replicas that were very close to historical percussion point locations, but only very rarely. I think that the people who build these are determined to produce a particularly authentic replica of a real sword, to which they have direct access. It doesn't seem to happen by chance.

Just for completeness, here's a chart showing direct control over balance point.

	METRIC				***ENGLISH***			
	full taper	1/10 taper	no taper		full taper	1/10 taper	no taper	
blade mass	0.750	0.750	0.750	kg	1.65	1.65	1.65	lbs
blade length	1.000	1.000	1.000	m	39.37	39.37	39.37	inches
COM location	0.333	0.400	0.500	m	13.12	15.75	19.69	inches
I_{COM}	0.042	0.055	0.063	$kg\text{-}m^2$	0.987	1.302	1.480	$lb\text{-}ft^2$
COM to tip	0.667	0.600	0.500	m	26.25	23.62	19.69	inches
handle length	0.203	0.203	0.203	m	8.00	8.00	8.00	inches
pommel mass	0.372	0.511	0.721	kg	0.82	1.12	1.59	lbs
ΔCOM	0.178	0.245	0.345	m	7.00	9.63	13.57	inches
new COM	0.155	0.155	0.155	m	6.12	6.12	6.12	inches
new I_{COM}	0.113	0.166	0.244	$kg\text{-}m^2$	2.682	3.922	5.785	$lb\text{-}ft^2$
new mass	1.122	1.261	1.471	kg	2.47	2.77	3.24	lbs
I_{FINGER}	0.140	0.196	0.280	$kg\text{-}m^2$	3.324	4.644	6.625131	$lb\text{-}ft^2$
Ihandle	0.163	0.215	0.294		3.855	5.096	6.957094	$lb\text{-}ft^2$
COP to COM	0.649	0.845	1.069	m	25.56	33.25	42.09	inches
tip to COP	0.195	0.000	-0.224	m	7.69	0.00	-8.84	inches
static moment	0.174	0.196	0.228	kg-m	15.106	16.986	19.791	inch-lbs

Notice that these swords don't vary as badly as the ones built purely to match static moment. Their weights don't vary as much, nor do their static moments. But their percussion point locations are still 8 to 9 inches off, both short and long. So trying to match up balance points gives somewhat more accurate results, but still leaves bad percussion point locations.

And to keep control of the percussion point location, within say ¾ inches either way, means you have to have good control of pommel mass. For the sword with 1/10th taper ratio, the pommel has to be within 3% of the correct mass, otherwise the percussion point drifts off the tip or falls back onto the blade. This is only a half-ounce, and on a pommel that weighs over a pound. If you have a sword that has a precisely controlled percussion point, it took precision to put it there.

Before I leave this subject entirely, let me show another spreadsheet that includes a blade that is complete non-tapering for 2/3 of its length, then tapers linearly to nothing in the final 1/3. This is what's been called distal taper, meaning the distal end of the sword has been tapered, while the rest is left straight. In the spreadsheet, this sword replaces the sword that had absolutely no mass taper, since no one would ever really make one of those.

	METRIC				ENGLISH			
	full taper	1/10 taper	tip taper		full taper	1/10 taper	tip taper	
blade mass	0.750	0.750	0.750	kg	1.65	1.65	1.65	lbs
blade length	1.000	1.000	1.000	m	39.37	39.37	39.37	inches
COM location	0.333	0.400	0.421	m	13.12	15.75	16.57	inches
I_{COM}	0.042	0.055	0.047	kg-m^2	0.987	1.302	1.105	lb-ft^2
COM to tip	0.667	0.600	0.579	m	26.25	23.62	22.80	inches
handle length	0.203	0.203	0.203	m	8.00	8.00	8.00	inches
pommel mass	0.511	0.511	0.557	kg	1.12	1.12	1.22	lbs
ΔCOM	0.217	0.245	0.266	m	8.56	9.63	10.47	inches
new COM	0.116	0.155	0.155	m	4.56	6.12	6.10	inches
new I_{COM}	0.129	0.166	0.171	kg-m^2	3.059	3.922	4.051	lb-ft^2
new mass	1.261	1.261	1.307	kg	2.77	2.77	2.87	lbs
I_{FINGER}	0.146	0.196	0.202	kg-m^2	3.460	4.644	4.794	lb-ft^2
Ihandle	0.165	0.215	0.220		3.899	5.096	5.220	lb-ft^2
COP to COM	0.884	0.845	0.845	m	34.81	33.25	33.27	inches
tip to COP	0.000	0.000	0.000	m	0.00	0.00	0.00	inches
static moment	0.146	0.196	0.202	kg-m	12.655	16.986	17.535	inch-lbs

As you can see from the data, the sword requires a heavier pommel, by about 9%. The final mass of the sword only goes up by 3.6%. Interestingly, the moment of inertia about the finger, or about the mid-point of the handle, is almost the same as the sword that taper's linearly, ending with 1/10th the mass/length at the tip, as it had at the hilt. The balance point also comes within 0.02 inches of the continually tapering model. Sure enough, as stated previously in the section on moment of inertia, it's the mass at the tip that kills the maneuverability.

Why Balance Points Vary On Authentic Swords

The range of balance points and resulting static moments that are found on authentic swords is not an artifact of the user's preferences. These variations are the natural result of a variety of blade types that have had their percussion point locations adjusted with a pommel. In my examinations of authentic Renaissance swords, a limited examination to be sure, I've found that they all have nearly identical percussion point locations, much unlike reproduction swords, whose percussion point locations are widespread, and apparently almost random. This tight clustering of percussion point locations can't be achieved through controlling balance point or static moment, as shown previously. It must be done carefully, and intentionally. We would be at a loss to explain the resulting variation in balance point and static moment without the insight into pommel and percussion point physics.

It's similar to understanding the balance point locations found on 20th century fighter aircraft. Some balance close to the front, about a third back from the nose, which is common in WWII aircraft, while many later jet fighters balance very far toward the back. The balance point doesn't correlate to weight or length. So when you build a reproduction fighter aircraft, can you choose any balance point you want? Not once you understand what the wing does. You have to balance your fighter just a bit in front of the wing's aerodynamic center. Just as any young boy today would be horrified at the thought of a model plane that didn't balance anywhere even close to the wing, knights of old would probably be horrified at some of our reproduction swords percussion point locations, especially ones whose percussoin point is almost two feet past the end of the tip. Yet we can buy those very types from some of our best manufacturers!

Chapter 8
Objects in Rotation and Translation

Physical Properties of an Object in Rotation and Translation

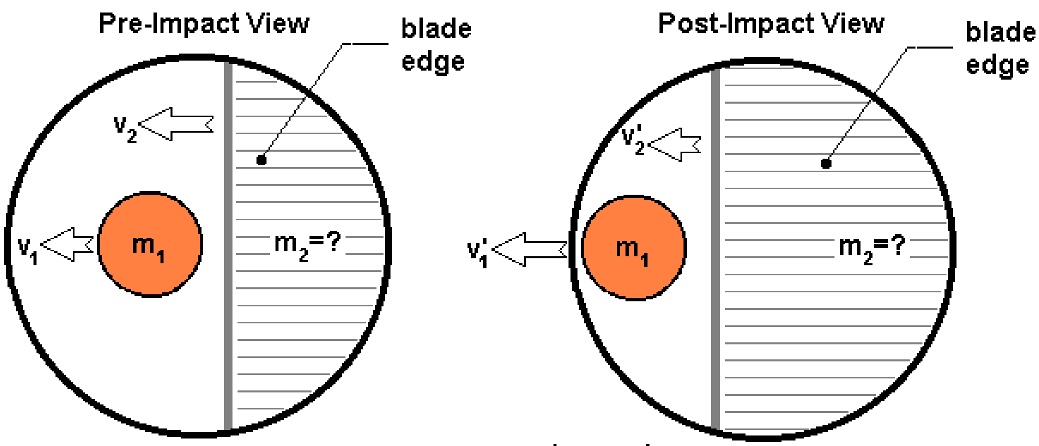
I came up with the odd realization about inertia after spending many months of frustration with more conventional approaches to sword impact theory. It gives the correct results, and much more simply than any existing method that I have managed to find. Unfortunately, I haven't found anything quite like it in my searches through physics and dynamics books, but it still remains the simplest approach I've tried. It of course also gives the correct results, and makes examining the impact behavior of various points along the edge rather easy. It also provides some useful insights of its own. However, I don't really know what to call the adjusted inertia that's used in the analysis. Hereafter, I will refer to it as either the apparent or perceived inertia of the sword. It is not a moment of inertia, just as a force is not a moment of force, which is termed a torque. It is simply the resistance of the sword to being pushed, as measured along the length of its edge. As a moment of force is a torque, which is a force at some distance, the resistance to rotation at some point, which we call the moment of inertia, would also be the inertia at some distance. I suspect Galileo or Sir Isaac Newton actually dealt with it this way, but don't have time to investigate further. But inertia is just resistance to an applied force, which is exactly what we're calculating. But if you're looking through textbooks on dynamics, you'll find inertia tensors, inertia dyadics, and other forms that use the term inertia, but never just plane old inertia. Well, here it is, and it's not the mass.

Inertia

In considering the impact of a sword, a useful insight can be gained by examining the results of an object that's rotating and translating, without considering the hand. One simplification can be gained by realizing that the target doesn't know the size and composition of the impacting object. The target object undergoing the sword's impact can only react as if another object with a particular mass and velocity had struck it. This allows us to simplify the model of impact behavior by reducing the sword edge to a perceived mass, traveling in a straight line, and this changes the impact model to a simple one of linear particle collisions. When your sword strikes a particle of mass m, that target particle will react as if struck by another particle. The velocity of the impacting particle is the same as the edge velocity of your sword at the impact point. By observing the initial and final velocities of both particles, and knowing the mass of the target particle, we can make a calculation as to mass of the impacting particle. Physicists do such calculations routinely when investigating atomic interactions in particle accelerators. However, our edge is connected to the rest of the sword, and this calculation comes up with a value that's always less than or equal to the true mass of the sword. What we're actually calculating is the inertia of the sword's edge at the impact point.

Microscopic View of an Impact

The inertia of the blade's edge is just the target mass times the ratio of the change of velocities.



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2)$$

$$m_2 = m_1 \frac{(v_1 - v_1')}{(v_2' - v_2)}$$

By inertia, I don't mean the actual mass, but rather the property of resisting acceleration. Currently, physicists always take inertia to be exactly equal to the mass, which is why it barely rates a single sentence in any modern physics book. But Galileo, Newton, and others always retained inertia as a separate concept. They could've simply said that all objects resist acceleration according to their weights, or masses, but to any swordsman that would've been completely refutable. A heavy sword with a small moment of inertia doesn't strike as hard as a light sword with a high moment of inertia. From Tycho Brahe losing his nose in a sword fight, to Rene Descartes doing physics as a sideline to soldiering, swordsmen invented much of modern physics, and sword impact dynamics isn't nearly as simple as colliding ball impact dynamics. Christiaan Huygens theoretical impact work was done with pendulums, and he communicated his results to Isaac Newton, who ingeniously found a way through the mess, and the true mass of the sword, and its change in velocity, is also a valid solution to the equation shown in the above illustration. You might say I'm just rigging up another mass that can solve the impact equation, given the sword's edge velocity at the impact point. You might also speculate that Newton couldn't have been oblivious to this really handy mathematical shortcut when he carefully avoided saying that objects resist applied forces according to their mass, and stuck with the phrase inertia. Interestingly, Galileo is usually given credit for developing the concept of inertia, but medieval philosophers had already anticipated him in the discovery of this principle. But they made many other unsupported conjectures, and Galileo made very few. Regardless, what we're dealing with here is truly a medieval concept in dynamics.

Derivation of Inertia Along a Blade

One might think that if we hold a sword horizontally and drop it on our hand, our hand feels the sword's full weight and speed. However, this is the case only if we drop the sword so that its center of mass lands on our hand. If we move away from the swords center of mass, then the sword rotates away, not delivering a full impact. The inertia of the sword, measured perpendicular to the blade, lessens as we move away from the center of mass. If we were trying to measure the mass of the sword in zero gravity, we would do so by applying a known force and observing the resultant acceleration. By using the formula $F = m \cdot a$, we would calculate the mass. We could also use a spring to push and pull on the sword, making it oscillate back and forth. The oscillation frequency would likewise allow us to determine the sword's mass. However, if we are not pushing in line with the sword's center of mass, then our results are erroneous. We end up making the sword rotate instead of just moving linearly back and forth, and we end up measuring a mixture of the sword's mass and moment of inertia. Our common sense applies here. If a horizontal wood waster were to fall on our heads, we know the tip wouldn't hurt as much as the balance point would. If you have one handy, try it. When the tip hits your head, the sword rotates rapidly around, the tip yields easily to your head, and the hilt just continues falling at your side, almost unimpeded.

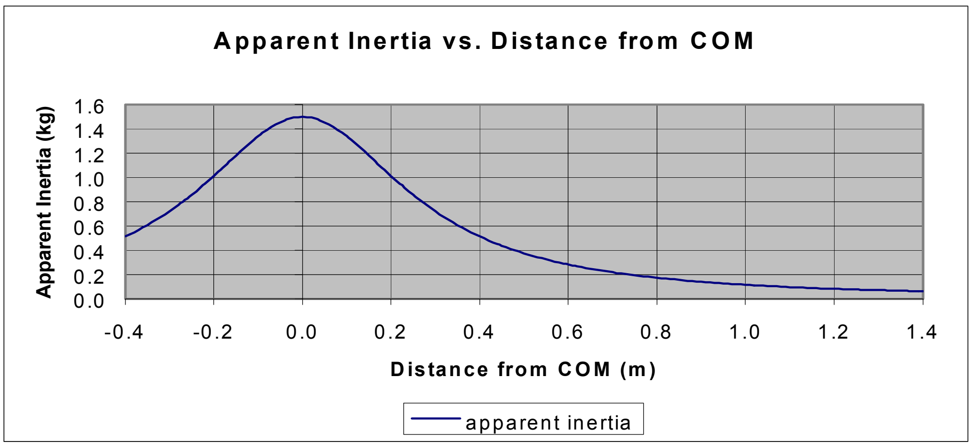
The apparent inertia we are dealing with is only measured in a line perpendicular to the direction, from the point being examined, to the sword's center of mass. Think of pushing on the very tip of the sword. Edge-to-edge, the inertia is pretty small. Side-to-side, the inertia might feel even smaller, due to blade flexure, but more on that later. However, if you push directly toward the sword's center of mass, as during a thrust, then the sword still seems to have all of it's mass present. You might think of the inertia as being measured in a particular direction, which forms a 3-D mapping of inertias. Since we're only presently concerned with delivering a blow, we can neglect the inertia in directions other than perpendicular to the edge.
What we are doing for this measurement is still just applying a force to some point on the sword, just as we've been doing in previous chapters to observe the resultant motion. But now we'll use these motions as an indicator of the sword's inertia at that point. So we'll examine the sum of the linear and tangential accelerations, which result from our applied force. Recalling our previous investigations, if we lay the sword horizontally, and define a coordinate system with $p = 0$ located at the swords center of mass, our question is how the sword reacts to forces applied vertically as we vary p, the point of force application.

Given a force F applied at point p, the center of mass reacts according to the equation $F = m \cdot a$, while the force also imparts a torque Γ according to $\Gamma = F \cdot p$. Since the sword reacts to the applied torque according to $\Gamma = I_{COM} \cdot \alpha$, and the tangential acceleration due to the angular acceleration α is given by $a_{TANGENTIAL} = \alpha \cdot p$, then the apparent linear acceleration at point p due purely to rotation is given by $a_{TANGENTIAL} = \frac{\Gamma}{I_{COM}} \cdot p$, or finally $a_{TANGENTIAL} = \frac{F \cdot p^2}{I_{COM}}$. Adding to this the linear acceleration of the center of mass, we get the total acceleration of point p due to force F, which is $a_{TOTAL} = \frac{F}{m} + \frac{F \cdot p^2}{I_{COM}}$. The perceived inertia, or resistance to acceleration, of the sword at point x is merely the applied force divided by the resultant total acceleration, or $m_{PERCEIVED}(p) = \frac{F}{\frac{F}{m} + \frac{F \cdot p^2}{I_{COM}}}$. Simplifying, we get

The equation for apparent inertia

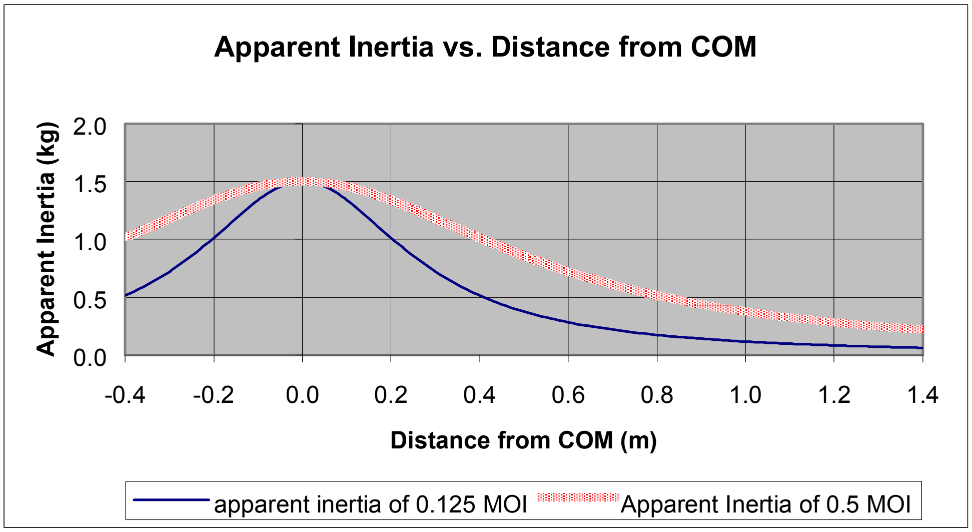
$$m_{PERCEIVED}(p) = \frac{1}{\frac{1}{m} + \frac{p^2}{I_{COM}}}.$$

This makes sense, as the perceived mass shouldn't depend on the applied force. Also, if we push in a direct line with the center of mass, then $p = 0$, so $\frac{p^2}{I_{COM}} = 0$, and $m_{PERCEIVED}(0) = \frac{1}{\frac{1}{m}} = m$. Additionally, as $p \to \infty$, $m_{PERCEIVED} \to 0$, meaning that when you've got an infinite leverage on the sword, it gives way to applied force so easily that you can't



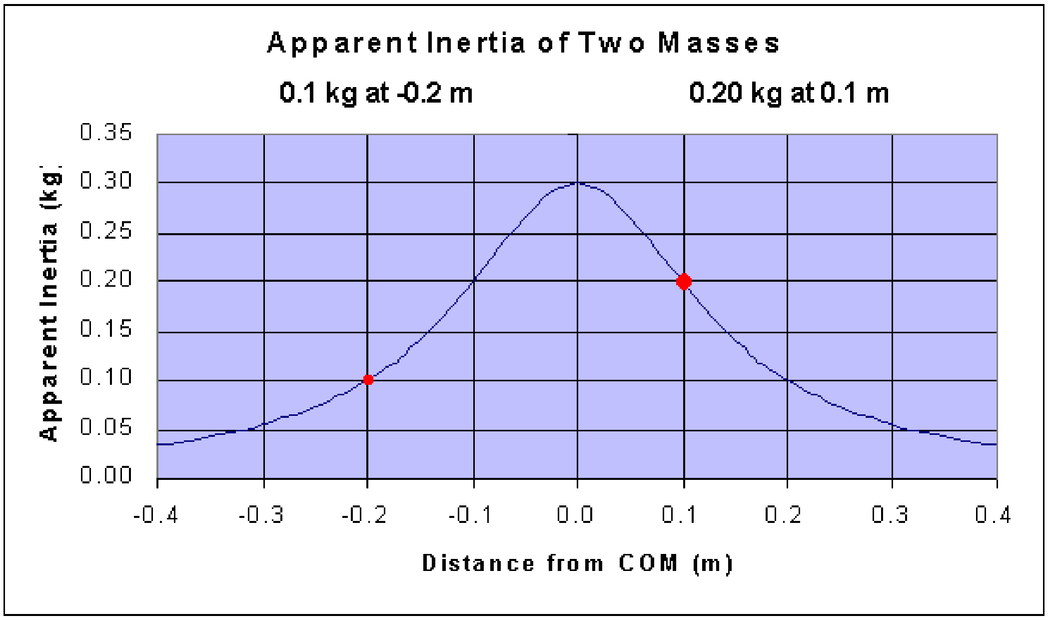
tell it's there. So an extended object, such as a sword, has a perceived mass that decreases as you move away from the mass's center. The units also work out, realizing that moment of inertia is just mass multiplied by distance squared. The term $\frac{p^2}{I_{COM}}$ can also be represented as $\frac{p^2}{m_r \cdot p_1^2}$, so the units of length cancel, leaving us with only the same units as used in the first part of the denominator, or $\frac{1}{mass}$. So the apparent mass can indeed be measured in kilograms, but by this we're not referring to a given quantity of matter, but merely an objects resistance to acceleration (in a particular direction), or inertia. When this perceived inertia is used during an impact analysis, the resulting motions of the target and sword edge do indeed yield a result that preserves the initial momentum of the sword and target system. This concept of perceived inertia allows us to analyze the results of collisions all down the edge of the blade, without crunching through all the numbers for the sword's final linear and angular velocities, applied torques, etc. It's all already included in the way apparent inertia was derived.

Graphically, for a 1.5 kg sword, with an $I_{COM} = 0.125 kg \cdot m^2$, the apparent mass appears as shown above. The curve is symmetric about the sword's center of mass, where the apparent mass exactly equals the true mass. Increasing the moment of inertia will spread the curve out, as seen in the following graph, where the moment of inertia has increased four fold. Also note that the apparent mass is not zero at the blade tip. In fact, the apparent mass never actual reaches zero, no matter how far out you go. Also note that we've come up with a sword with 4 times the moment of inertia as the original, so we might have to work much harder to swing it, possibly by a factor of 4 or more, depending on where the added mass is located, relative to the hand. Yet we get no benefit in apparent inertia near the center of mass, and have only got a 3.5 fold improvement out at the point 1.4 meters from the center of mass.



In the equation for inertia, $m_{PERCEIVED}(x) = \dfrac{1}{\dfrac{1}{m} + \dfrac{p^2}{I_{COM}}}$, we can think of the blade as consisting of a collection of small masses, as if we'd sawed it into small sections. Now look at just one small part of the sword, with a mass given as m_i. If this mass is out at some distance x from the center of mass, then the moment of inertia of the sword is at least as much as must be caused by the presence of the mass, plus at least one other mass that causes our mass to be somewhere other than the center of mass. We'll call this other mass m_j. Even if we have only these two masses, we can calculate the center of mass location, moment of inertia, etc. I'll spare you the math, but the result is simple. The apparent inertia at any point that mass a mass m_i is equal to or greater than the mass present at that location. If you have a single mass, the apparent inertia at the center of mass is just the mass. If you have two masses, the apparent inertia at each of those masses is just the mass present at each of those points. If you have many masses, the apparent inertia at one of those masses is always greater than or equal to the mass at that point. For all complex objects, the apparent inertia at some point on the object is always greater than the actual mass located at that point. Apparent inertia also reflects how much the mass at the impact point is "stiffened" by being solidly connected to the rest of the object.

Here's an example of two point masses, and their apparent inertia.

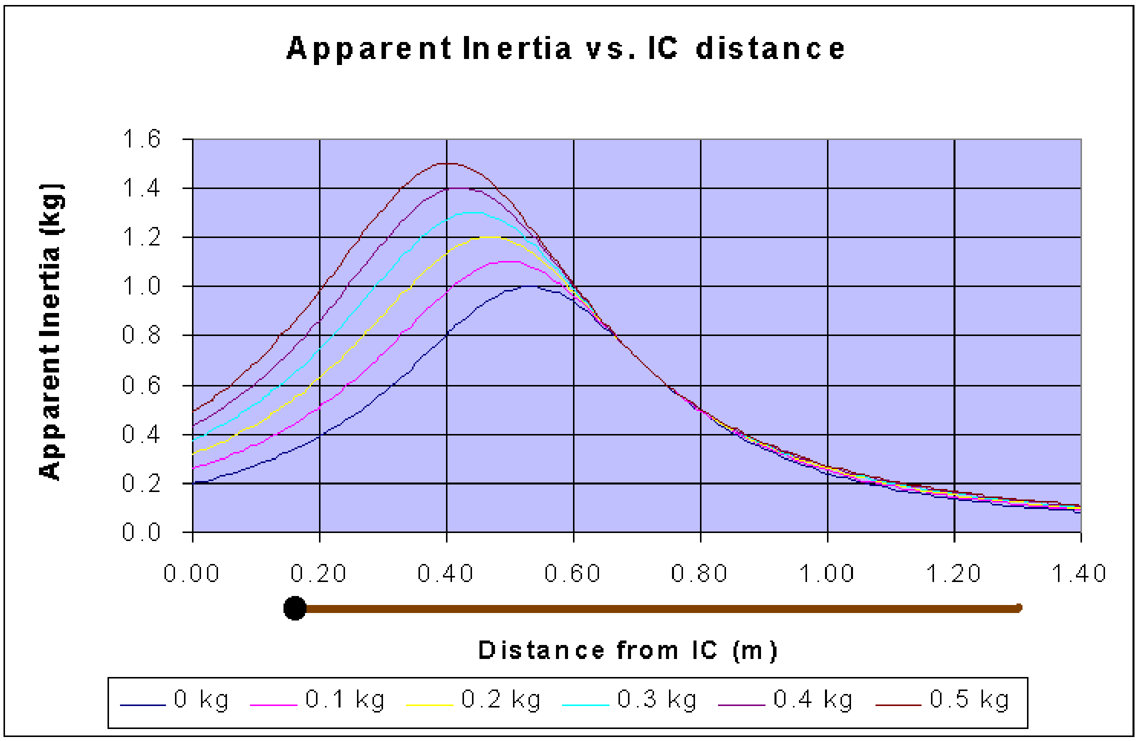


We can also show that the inertia can never exceed the mass, by rearranging the equation to solve for the moment of inertia. By taking our equation for inertia, or perceived mass $m_{PERCEIVED}(p) = \frac{1}{\frac{1}{m} + \frac{p^2}{I_{COM}}}$, and flipping the terms we get $I_{COM} = \frac{p^2}{1 - \frac{m_{PERCEIVED}(p)}{m}}$. If the perceived mass, or inertia, were greater than the object's true mass at any point x, the denominator in the equation would become negative, and we'd have a negative moment of inertia, which isn't physically possible. This also gives us an insight into another possible method of finding moments of inertia, just by using a collision to measure the inertia of some point on the blade.

The Pommel and Apparent Inertia

Let's switch gears, change the basis for the coordinate systems, and see how pommels affect this curve. Since making changes to the pommel's mass shifts the COM location, using the COM location as the basis of the x axis would skew the curves. You don't change your grip location when you change pommels, nor does your blade tip location shift. So let's pick some arbitrary point that doesn't shift around with pommel mass as the basis for the x axis. For this graph I'll use the instantaneous center of rotation, or IC, taken to be 0.15 meters inside of the pommel, based on baseball research.

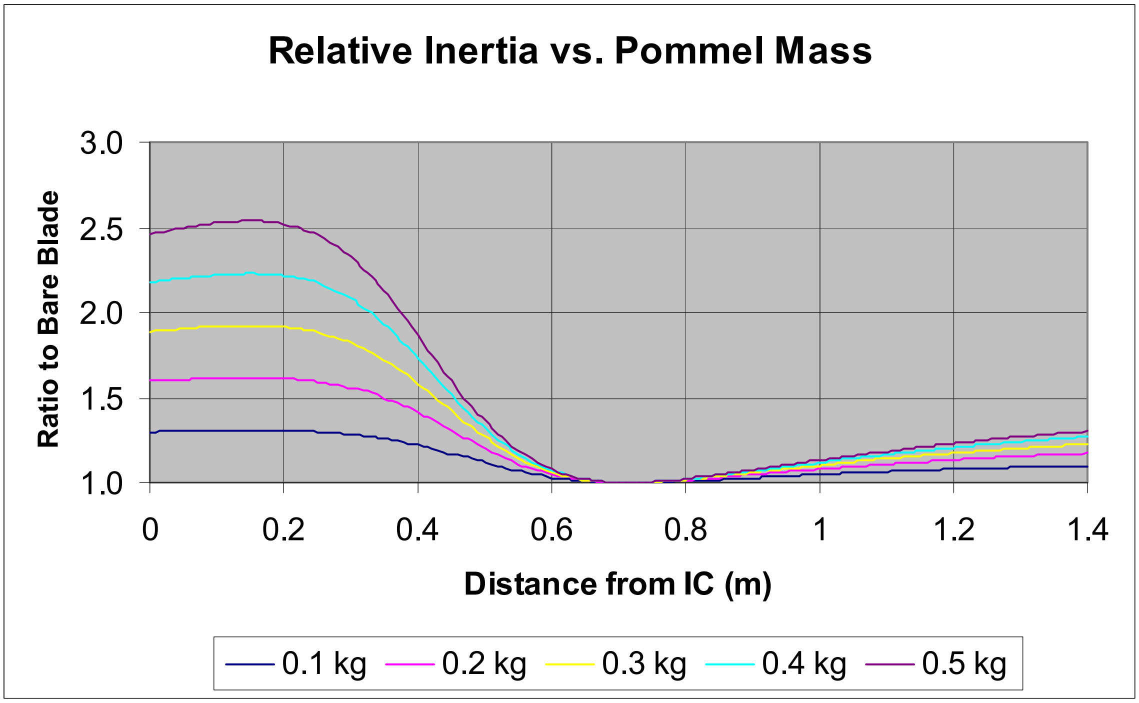
Here we've picked a bare blade, and then added five different pommels to it, and plotted the results. The brown stick represents the sword, with the pommel being added 0.15 meters from the swing's center of rotation.



The lowest curve represents the apparent inertia of the bare blade, along the blades length. Each successively higher curve represents a progressively heavier pommel. The peaks of the successive curves keep moving to the left, which represents the shift in the center of mass location. That's why the stick doesn't have an illustrated COM location, because the location shifts with each added pommel mass. The peaks keep getting higher because the sword's final weight keeps getting heavier as pommel mass is added. The biggest change occurs in the apparent inertia near the hilt. If you were using the sword as a club, hitting your opponent with the pommel in what is called a murder stroke, then you can see that heavier pommels dramatically increase the apparent inertia that impacts into your opponent's head. However, on out on the blade, the difference looks small.

You can also note that there are absolutely no bumps on the curve. You make some point on the blade carry an extra amount of apparent inertia. Welding a small mass to your sword's sweet spot won't make that area feel 'heavier' to the target. It will certainly add a great deal to your moment of inertia, since it's being added far from your hand, and the sword's center of mass, but other than flattening out the curve somewhat, it won't make any sort of peak out at the tip.

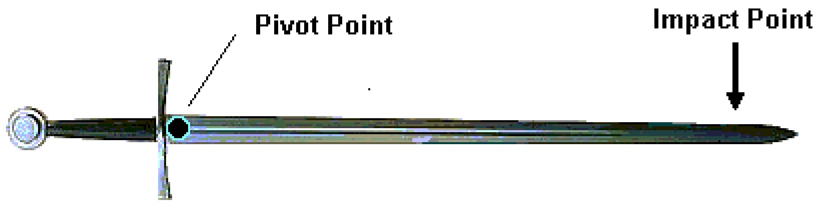
Let's now look at the apparent inertia compared to the apparent inertia of a bare blade, that doesn't have a pommel, to see how much improvement in relative inertia the pommel adds.



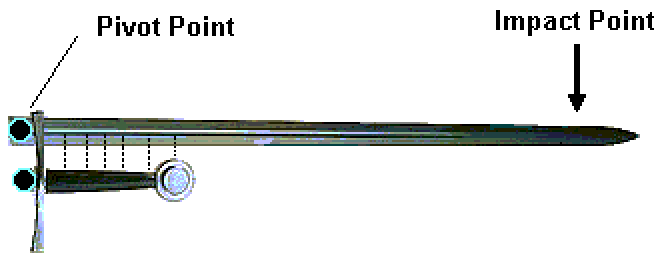
Here we can see that the pommel is greatly increasing the apparent inertia in the hilt area. It's also adding significant inertia as we approach the tip, in this case up by 30% with the 0.5 kg pommel. At no point does the curve dip below 1, which brings up another rule. Adding mass anywhere on the sword will never decrease the apparent inertia of any other point on the sword. We also have a mysterious point, in this case located at around 0.7 meters from the IC, which seems to have its apparent inertia entirely unaffected by the added pommel mass. What point would be unaffected by added pommel mass, no matter how much mass is added? That would be the percussion point relative to the pommel. In an impact on the pommel's percussion point, the change in rotation and linear motion of the sword acts to rotate *around* the pommel, so the mass of the pommel is irrelevant to the outcome of the impact. The apparent inertia of the percussion point relative to the pommel, is unaffected by the pommels mass. This brings up another important point. Adding mass to the pommel increases the apparent inertia of all points on the blade, except the percussion point relative to the pommel.

Amazingly, the blade tip has been "stiffened" 30% by a pommel mass that's nowhere near the tip. The 0.5 kg pommel increased I_{COM} by 69%, but since the center of mass has approached the hand, the moment of inertia about the hand has only gone up by 14%, so the increase in apparent inertia at the tip has been twice the increase in moment of inertia about the hand. That's a pretty neat trick. During an impact, as far as the target is concerned, the pommel is making the tip of the blade more "massive" or stiffer!

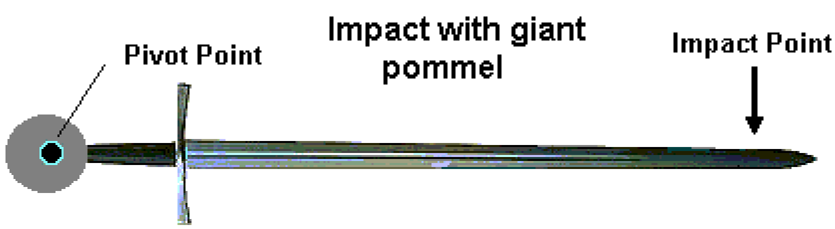
One way to understand this is to go back to how a sword rotates. We know the impact point, the moment of inertia, the mass, and thus the impact point's percussion point. This percussion point, relative to the impact point, is where the impact makes the sword pivot. The impact causes no new motion at the impact's percussion point, so you can consider this point to be permanently fixed.



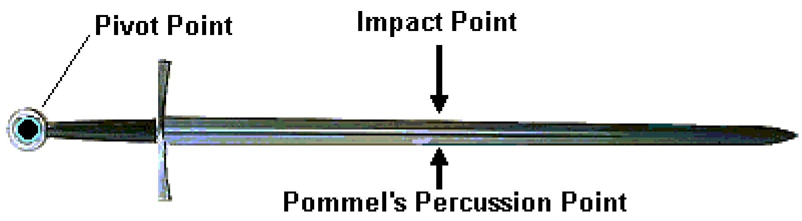
The impact causes the sword to rotate around this point. Now look where the pommel is located. It's a large mass that's on the other end of the sword, generally far from this pivot point. When you impact the sword's edge, the entire sword, including the pommel on the other side of the pivot point, is resisting the resulting rotation. The increase in required torque to rotate the sword is the same as if the hilt where folded over at the percussion point, so the tang and pommel where actually mounted out on the blade.



You might have a flash of inspiration and go for a giant pommel, but this doesn't work. It shifts the axis of rotation so close to the pommel that the sword can't be swung. Even just going with an overly large pommel just moves the percussion point out so far that the sword loses its tendency to realign itself during a swing.

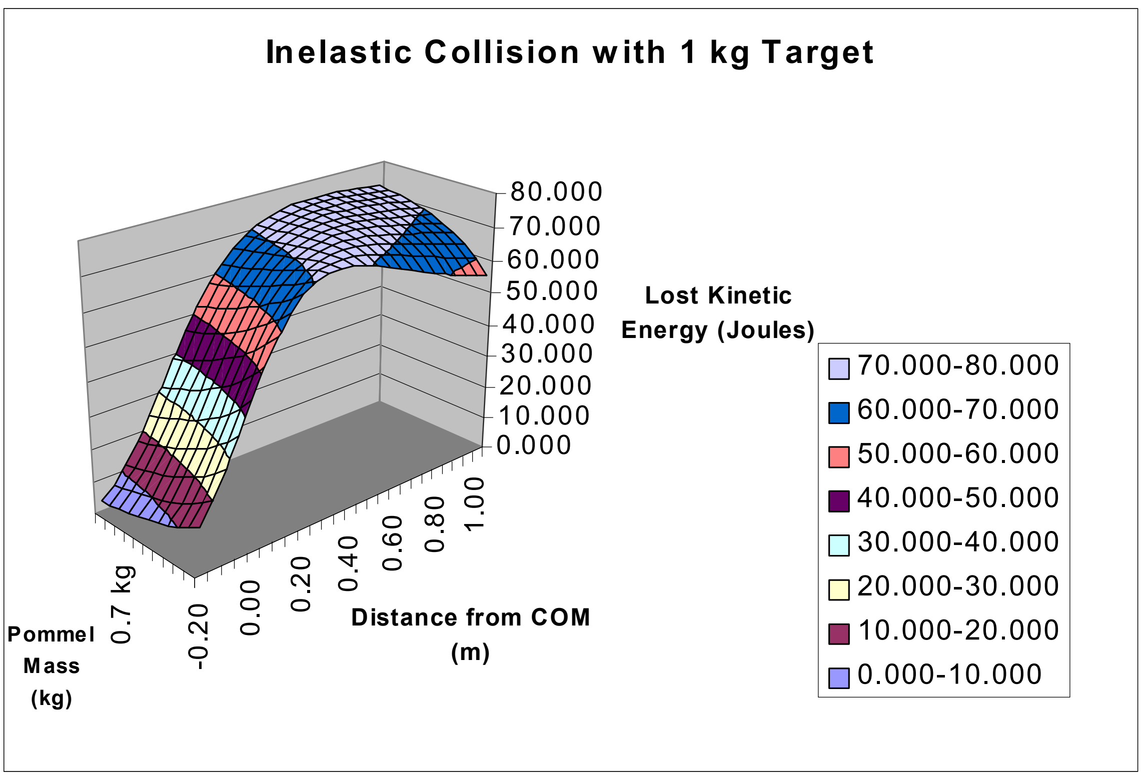


However, this also tells us something else. If you strike with the part of the blade that also happens to be the pommel's percussion point, which should be somewhere near the middle of the blade, that impact pivots around the pommel, as shown in the following figure.



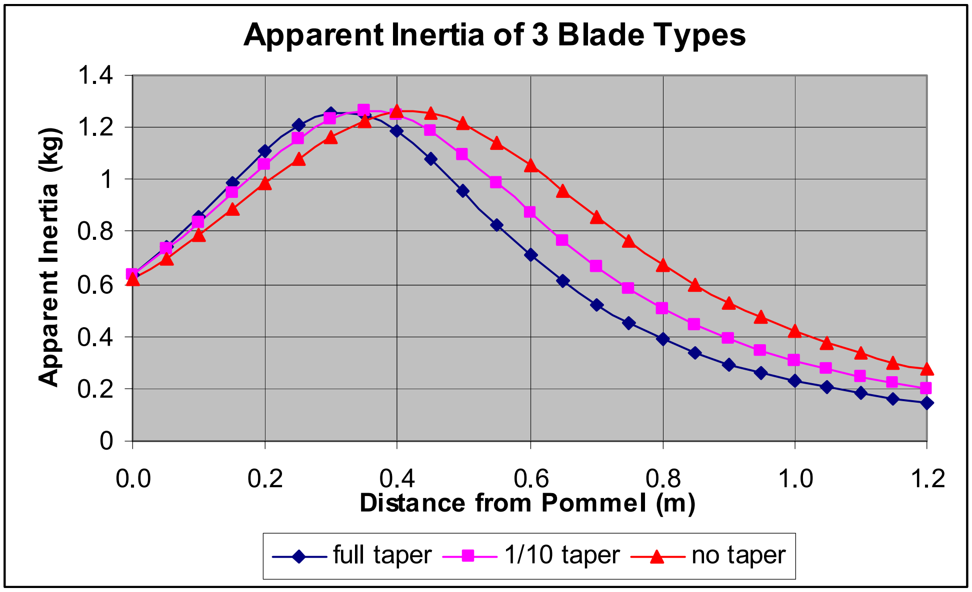
In that case, the pommel isn't doing anything to improve the impact characteristics. If that was the sword smith's desired impact point, he wouldn't have added a pommel, as far as impact dynamics are concerned. Striking with the pommel's percussion point, though it does some damage, doesn't benefit in any way from the presence of the pommel. Since the pommel also inhibits the swing, although only to a very small degree, impacts with the pommel's percussion point are degraded by the presence of the pommel. Obviously, that area of the blade is probably not, in general, the preferred zone to strike with.

To sum up, here's a graph of an impact, with a 1-kg inelastic target, with varying masses for the pommel. In front, the sword has no pommel, and impact damage trails off toward the tip. For each line in the graph, moving toward the back, the pommel mass has been increased by 0.1 kg. At the back, you can see that the high impact damage, in this particular setup, is maintained all the way to the blade's tip.



Effect of Blade Taper on Apparent Inertia

At the end of the previous chapter, we compared blades of various tapers, illustrating the fact that they all required exactly the same pommel. In the very first spreadsheet that I included, I compared a fully tapered blade, with blade whose tip tapered to 1/10 the mass of the hilt, and a blade that didn't taper at all. Here is a graph of the apparent inertia of those same blades.



You can see that for all positions past the tip-heavy blade's percussion point, the apparent inertia is much greater for the tip-heavy blade than it is for the more maneuverable blades. Interestingly, the non-tapered blade, compared to the fully tapered blade, has exactly 85.55% more apparent inertia at the tip, which is the percussion point on these blades. More interestingly, its moment of inertia about the forefinger, which is the other percussion point, is exactly 85.55% higher than on the fully tapered blade. This actually is expected, since the apparent inertia is a measure of how hard it is to move the blade, when pushed at a particular point. Since whatever point we push has a corresponding percussion point, which may or may not be on the blade, we are merely rotating around that percussion point. For identical distances to that percussion point, the varying moment of inertia of the difference blades, around that percussion point, is an exact reflection of the apparent inertia in the blow. So for blades that have the same percussion point distance, how hard the blade will impact, when struck on your hands percussion point, is exactly related to how hard the blade is to maneuver. That rotational heaviness, which makes the blade difficult to maneuver, is exactly what you're using to inflict damage on your target.

The tip-heavy blades make strong strikes. The really light blades make weak strikes. This is what we already intuitively know. It's just the mathematical reason that an edged foil isn't likely to dent any armor, or dismember anyone. It's also the reason you can use a big tip-heavy blade, that can't maneuver well, to cut firewood. However, as we make the percussion point distance longer, the apparent inertia gets less, as long as we keep the same moment of inertia about the impact point's percussion point. You can see that from the downward slope on the graph of the apparent inertia of the differing blades. In changing the percussion point distance, you're still just playing the game of trading off mass for velocity, since points further out will also be moving faster.

The reason for this is really simple. You apply some force F to the blade, which causes the blade to pivot around the corresponding percussion point, located at some position k from where you apply the force. The sword then pivots around this point. The force also applies a torque, given as $\Gamma = F \cdot k$. But we also know that the torque causes an angular acceleration about point k, given as $\Gamma = I_K \cdot \alpha$, where I_K is the moment of inertia around the pivot point. So we have $F \cdot k = I_K \cdot \alpha$. The linear component of acceleration of the point you push is simply given as $a = \alpha \cdot k$, so we can substitute in for the angular acceleration with $\alpha = \frac{a}{k}$, giving us $F = \frac{I_K}{k^2} \cdot a$. This is simply Newton's law $F = m \cdot a$, where the mass m has been replaced by the moment of inertia about the percussion point, divided by the square of the distance to that percussion point. So we're pretending that $m = \frac{I_K}{k^2}$. This is an alternate, but equivalent, formulation of apparent inertia. How hard the blade is to push, at some particular point, is directly measured by the moment of inertia measured around that point's corresponding percussion point, divided by the square of the distance to that percussion point.

Simple Calculation of a Sword's Tip Inertia

Given the equation for apparent inertia, we might wonder what it would be at the tip of our sword, since that is generally near our desired impact point. Given that the tip of the sword is the percussion point relative to the forefinger, if the pommel has been correctly sized, we make some substitutions into the equation for apparent inertia. Since our impact point will be the tip, which is the percussion point, we can subsitute Q for p in the equation

$m_{PERCEIVED}(p) = \frac{1}{\frac{1}{m} + \frac{p^2}{I_{COM}}}$. But from the percussion point equation we also know that

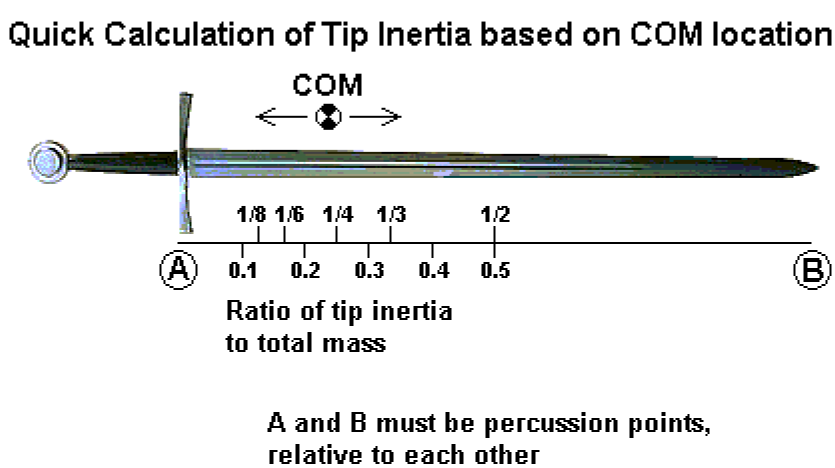
$I_{COM} = Q \cdot x \cdot m$, so we get $m_{PERCEIVED}(Q) = \frac{1}{\frac{1}{m} + \frac{Q^2}{Q \cdot x \cdot m}}$. To this we can multiply both the

numerator and denominator by m, and cancel the Q's, giving $m_{PERCEIVED}(Q) = \frac{m}{1 + \frac{Q}{x}}$, which

is a major simplification. But going further, we can get $m_{PERCEIVED}(Q) = \frac{m}{\frac{x}{x} + \frac{Q}{x}}$,

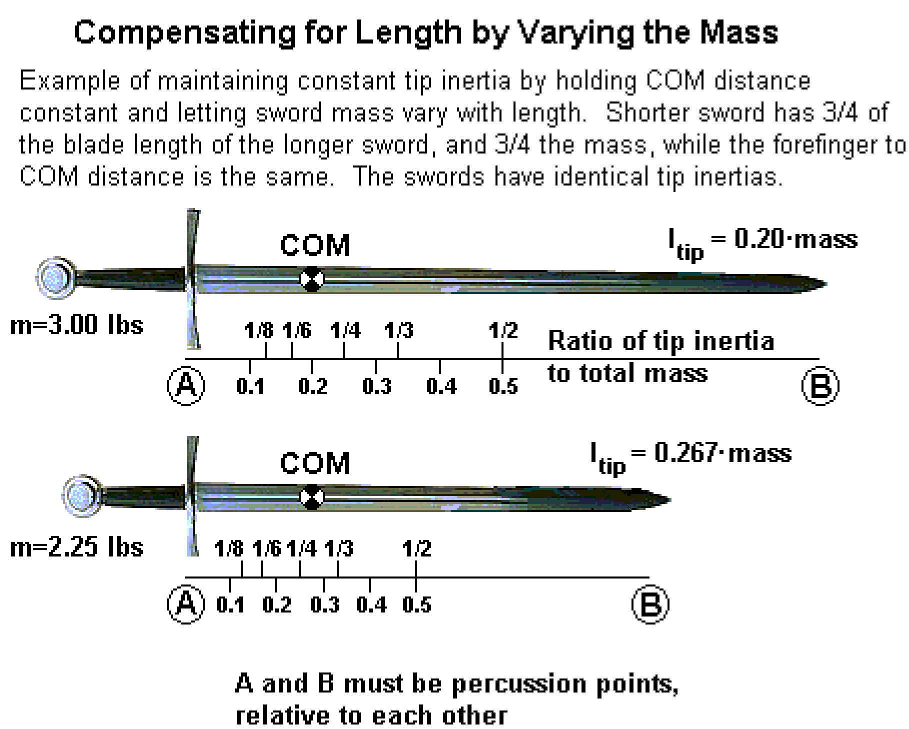
$m_{PERCEIVED}(Q) = \frac{m}{\frac{Q+x}{x}}$, and $m_{PERCEIVED}(Q) = m \cdot \frac{x}{Q+x}$. But if Q is the distance from the sword tip to the sword's COM, and x is the distance from the sword's COM to the forefinger, then the sum of these is simply the blade length. This leaves us with the incredibly simple equation

$m_{PERCEIVED}(Q) = m \cdot \frac{x}{L}$. So figuring out the tip inertia of a proper sword is trivial. It's simply the ratio of the balance point distance to the blade length, multiplied by the mass. If you have a 1 kg sword that balances ¼ of the way down the blade, the tip inertia is ¼ kg. If it balances one fifth of the way down the blade then the tip inertia is 0.2 kg.



This simple result would probably be known to anyone who used swords, could do simple ratios between objects of like kind, and knew how to weigh things. This would describe every educated person ever since the Greeks studied geometry and ratios. Interestingly, under Euclid's definition of ratio, only things of a like kind could form a true ratio. So medieval swordmen or philosphers could make a ratio comparing a distance to a distance, or a velocity to a velocity, but couldn't make a ratio between a distance and a velocity. This inhibited Galileo a bit, but fortunately for us another swordsman, Rene Descartes, brought algebra into geometry and fixed this lapse. So even under this limitation, comparing a ratio of balance point to blade length is possible. There's no theoretical or practical reason why anyone couldn't have calculated exactly how hard a particular sword's tip would strike, just by weighing it and finding the ratio of balance point to blade length.

This lets us easily compare different blades, or even keep the tip inertia constant as we vary the length or mass, just by using blades that naturally produce different balance points. Given the equation $m_{PERCEIVED}(Q) = m \cdot \frac{x}{L}$, we can see that if we wanted to match another equally heavy sword's tip inertia, but the length varied, we can compensate by adjusting the balance point. If the masses were allowed to differ, we could rewrite the equation as $m_{PERCEIVED}(Q) = x \cdot \frac{m}{L}$, so we hold the mass to length ratio constant, and also keep the distance to the center of mass a constant, for example always holding it as 12 cm and letting mass go up linearly with blade length.



Be very aware that you can't adjust the sword's center of mass location by simply adjusting the pommel. That would through off the percussion point, and ruin the sword's rotational dynamics. In fact, the pommel will likely have the opposite of the intended effect, since an attempt to reduce the sword's tip inertia and increase its maneuverability requires the center of mass location that's closer to the hand. Adding a pommel will move the COM location, but slightly decrease the maneuverability, and increase the tip inertia. To adjust the center of mass, you have to adjust the blade itself.

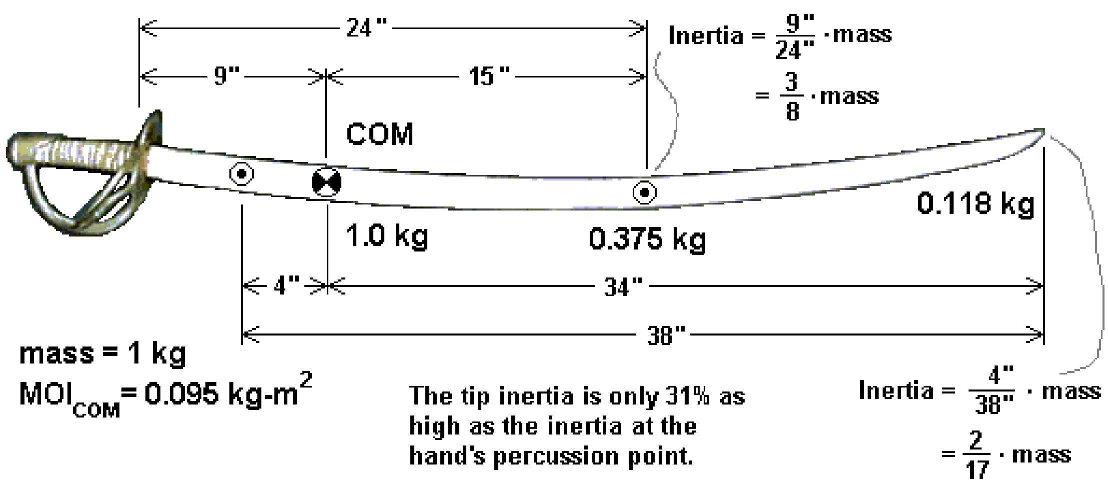
If you want to calculate the inertia at some point p, other than the hand's natural percussion point, the method is equally simple. Push on the blade at point p to find its corresponding center of percussion, somewhere down near the hilt. The inertia at that point will simply be the total mass of the sword multiplied by the ratio of *a* to *b*, where:

a) The distance from the sword's center of mass to point p's center of percussion.
b) The distance between point p and its percussion point.

For example, with my Austrian cavalry saber, the blade tip to forefinger distance is about 43 inches, and the balance point is about 9 inches from the forefinger. This is about 21% of the distance from the forefinger to the blade tip. You might naively think implies quite a bit of tip inertia in a sword that weighs 1.108 kg, or 2.4 lbs. But the actual percussion point is very far from the tip, only 24 inches from the forefinger. This implies a much larger inertia, but this inertia is at the percussion point, far from the tip. To actually find the tip inertia, we have to find the percussion point relative to the tip, which is only 4 inches from the sword's center of mass, or 5 inches in front of the forefinger. So x is much smaller, by more than half. Yet we also have the much greater length, from the tip of the sword to the center of mass, not the much shorter length from the forefinger's relative percussion point.

Diagram of a Cavalry Saber's Inertia

Inertia at the hand's percussion point vs. inertia at the sword's tip



So the saber has lower tip inertia, yet higher static moment. When you pick one up, the static feel tells you that it will do great damage, but be hard to wield. The static feel is deceiving. The tip does less damage than a more balanced sword, yet the saber feels heavier. This is all do to its percussion point being far back on the blade, and the center of mass being further out.

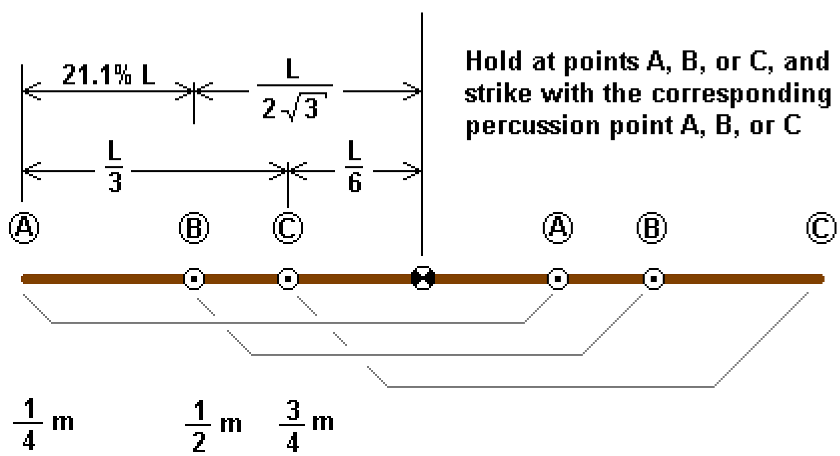
Calculation of the Inertia at Various Points on a Staff

For a simple untapered staff, the inertia at key points is also easily determined. From the inertia equation we know $m_{PERCEIVED}(p) = \dfrac{1}{\dfrac{1}{m} + \dfrac{p^2}{I_{COM}}}$. But for an untapered staff the inertia about the center of mass is given by the formula $I_{COM} = \dfrac{m \cdot L^2}{12}$. Substituting in, we get $m_{PERCEIVED}(p) = \dfrac{1}{\dfrac{1}{m} + \dfrac{12 \cdot p^2}{m \cdot L^2}}$, and multiplying top and bottom by the mass we have $m_{PERCEIVED}(p) = \dfrac{m}{1 + \dfrac{12 \cdot p^2}{L^2}}$. However, obviously any point p is going to be some fraction of L, so the whole equation will always simplify itself to a fractional mass. For example, at the tip, $p = \dfrac{1}{2} \cdot L$, so $p^2 = \dfrac{1}{4} \cdot L^2$, and $12p^2 = 3L^2$, and so $m_{PERCEIVED}(\dfrac{L}{2}) = \dfrac{m \cdot L^2}{L^2 + 3L^2}$, or $m_{PERCEIVED}(\dfrac{L}{2}) = \dfrac{m \cdot L^2}{4L^2}$, and finally $m_{PERCEIVED}(\dfrac{L}{2}) = \dfrac{1}{4} \cdot m$.

At the point 1/3 back from the tip, which is the percussion point relative to the other tip, p is just one sixth of the length from the center of mass, so $p = \dfrac{1}{6} \cdot L$, so $p^2 = \dfrac{1}{36} \cdot L^2$, and $12p^2 = \dfrac{L^2}{3}$. So $m_{PERCEIVED}(\dfrac{L}{6}) = \dfrac{m \cdot L^2}{L^2 + \dfrac{L^2}{3}}$, $m_{PERCEIVED}(\dfrac{L}{6}) = \dfrac{m \cdot L^2}{\dfrac{3L^2}{3} + \dfrac{L^2}{3}}$, $m_{PERCEIVED}(\dfrac{L}{6}) = \dfrac{m \cdot L^2}{\dfrac{4L^2}{3}}$, $m_{PERCEIVED}(\dfrac{L}{6}) = \dfrac{m}{\dfrac{4}{3}}$, and finally $m_{PERCEIVED}(\dfrac{L}{6}) = \dfrac{3}{4} \cdot m$.

So we have the handy result that for all untapered staff weapons, the tip inertia is ¼ the mass, and if we hold the staff by the tip, to strike with the point 1/3 back from the other tip, the inertia is ¾ the mass. This holds for all uniform and untapered objects, and is worth remembering.

But no simple fractional ratios produce the ½ mass point, and thisis worth some study. If we hold that the inertia, according to the equation $m_{PERCEIVED}(p) = \dfrac{1}{\dfrac{1}{m} + \dfrac{p^2}{I_{COM}}}$ is simply one half of the mass, then we have $\dfrac{1}{2} \cdot m = \dfrac{1}{\dfrac{1}{m} + \dfrac{p^2}{I_{COM}}}$, multiplying both top and bottom by the mass, we get $\frac{1}{2}\cdot m = \frac{m}{1+\frac{m\cdot p^2}{I_{COM}}}$. But realizing that the percussion point equation tells us that $I_{COM} = Q\cdot m\cdot x$, we can substitute and get $\frac{1}{2}\cdot m = \frac{m}{1+\frac{m\cdot p^2}{Q\cdot m\cdot x}}$. This simplifies to $\frac{1}{2}\cdot m = \frac{m}{1+\frac{p^2}{Q\cdot x}}$, which further simplifies to $\frac{1}{2} = \frac{1}{1+\frac{p^2}{Q\cdot x}}$. Applying yet more algebra, by flipping the equation over, we get $2 = 1+\frac{p^2}{Q\cdot x}$, and subtracting one from both sides gives us $1 = \frac{p^2}{Q\cdot x}$, and finally $p^2 = Q\cdot x$. Since $Q\cdot x = \frac{I_{COM}}{m}$, which is the moment of inertia to mass ratio of the weapon, which is a constant, so $p^{\cdot 2}$ must be a constant, and thus $p = \pm\sqrt{\frac{I_{COM}}{m}}$. This obviously has two solutions, but the solutions are equidistant from the object's center of mass, given simple algebra. So any object has an inertia that's equal to exactly half of its mass at the pair of percussion points that are equidistant from its center of mass, given by the square root of the moment of inertia to mass ratio. For a simple untapered staff, this is given as the square root of the moment of inertia to mass ratio. But since the untapered staff has a moment of inertia, around the center of mass, given by $I_{COM} = \frac{m\cdot L^2}{12}$, the equation for point p is just $p = \pm\sqrt{\frac{I_{COM}}{m}} = \pm\sqrt{\frac{m\cdot L^2}{12\cdot m}}$, which is just $p = \pm\sqrt{\frac{L^2}{12}}$, or $p = \pm\frac{L}{2\sqrt{3}}$.. If we evaluatethis numerically we find that $p = 0.288675\cdot L$ from the center of mass, or $p = 21.132485\%$ back from either tip. So, roughly speaking, if you hold an untapered weapon about 1/5 from one end, and strike about 1/5 from the other end, you are striking with about half the total mass of the weapon, and striking with the percussion point relative to your hand.



Sword Edge Velocity and Apparent Momentum

If we merely move the sword in translation, as if we were shoving it edge-on toward our opponent, then each point along the blade would have the same velocity. The apparent momentum and apparent energy curves, since edge velocity is a constant along the x-axis, would have the same profile as the apparent mass distribution, as shown above. So if you toss a sword from a moving car, the best impact point to cause maximum damage to a mailbox would be at the sword's center of mass. The mistaken notion that the center of percussion is the point on the blade where the force of the blow is consentrated, which is still the Oxford English Dictionary definition, also leaves us some interesting historical records. From the OED we get this nice citation.

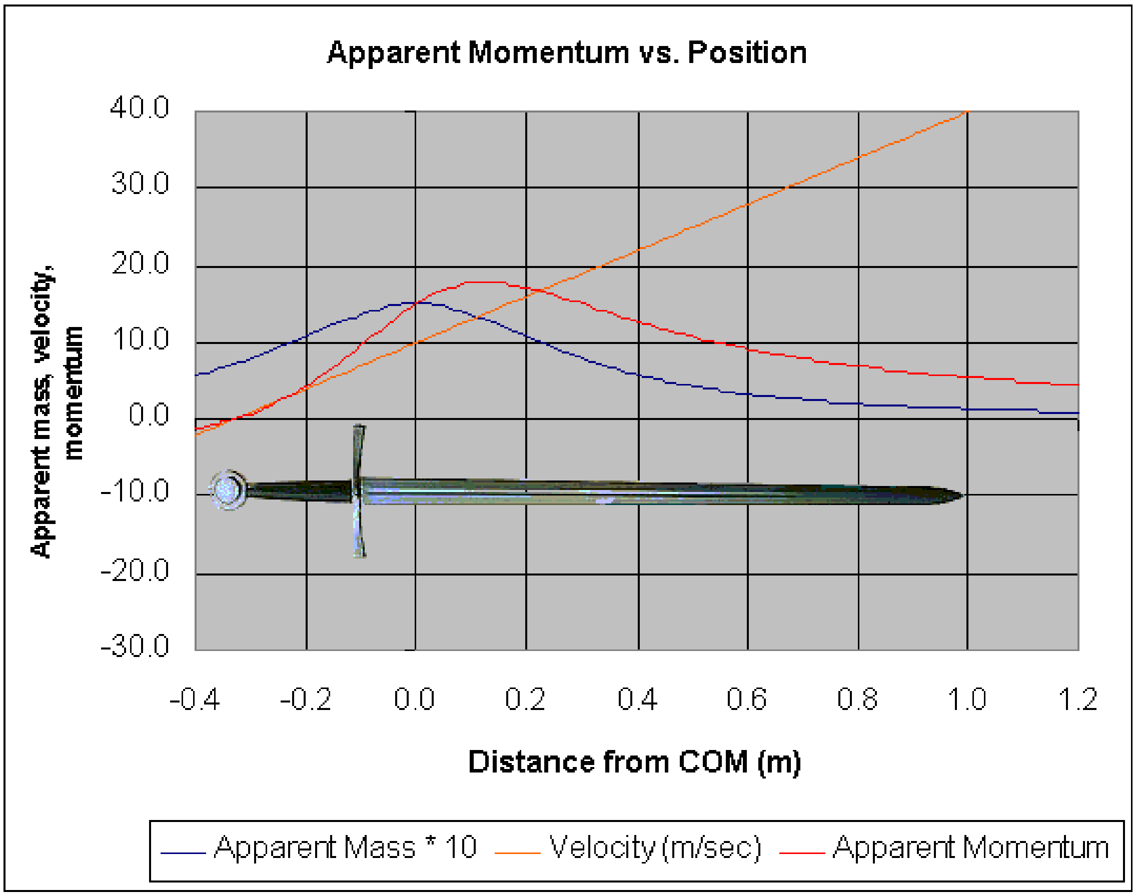
1726-51 Chambers *Cycl*. s.v. "The center of percussion is the same with the center of gravity, if all parts of the percutient body be carried with parallel motion"

Obviously, I'm not the first person to come up with this simple bit of physics. Also of interest is the alternate use of the term *center of percussion*, and the early date at which it appears. The current dictionary definition of the term refers to the point where all of the force of the blow is struck. Aside from realizing that all the force is concentrated at the impact point, which isn't very helpful, such a point doesn't exist. If all the force is consentrated at one point on the edge, then there remains no force anywhere other than this point, since the operative word is *all*. You could rework the definition to be most of the force, or the maximum force, which was the sense used in the above definition. However, you can't calculate this point till you know the mass and velocity of the target, plus the elasticity of the resulting impact, on top of the linear and angular velocity of the sword. This concept that the force is somehow concentrated in a particular spot is simply misleading. The energy in the blade can't be wandering up and down, trying to anticipate the impact point, and guessing the target's properties, so as to concentrate itself for the most efficient stroke. So I reject this alternate definition as worthless, since it doesn't refer to any constant properties of an object, and must be calculated on a case-by-case basis, for each different blow. There is a point in each blow where the maximum damage will be done, but we might as well refer to it as the peak of the damage curve, point of maximum damage, or some other term.

However, let's leave the insight about a sword carried in parallel, or non-rotating motion, and return to figuring out the rotating blade. If we swing the sword, then the edge velocity increases linearly, from zero at the instantaneous center of rotation, to a value given by $v(z) = \omega \cdot z$, where z is the distance from the IC and ω is the sword's angular velocity. We can simplify this by noting that the sword's center of mass has a tangential velocity given by $v_{COM} = \omega \cdot x_{IC}$, where x_{IC} is the distance from the swing's IC to the sword's COM. The velocity at any point on the edge can be described by $v(x) = v_{COM} + \omega \cdot x$, where x is the distance from the point on the edge to the sword's center of mass. This means that a line, which has a slope given by ω and a y-intercept at $y = v_{COM}$, describes the velocity profile of the edge.

If we multiply this sloping line by the apparent mass profile, we get an odd function that describes the apparent momentum of the points along the sword's edge. This is given by the equation $momentum_{PERCEIVED}(x) = m_{PERCEIVED}(x) \cdot v(x)$, or

$$momentum_{PERCEIVED}(x) = \left(\frac{1}{\frac{1}{m} + \frac{x^2}{I_{COM}}} \right) \cdot (v_{COM} + \omega \cdot x).$$

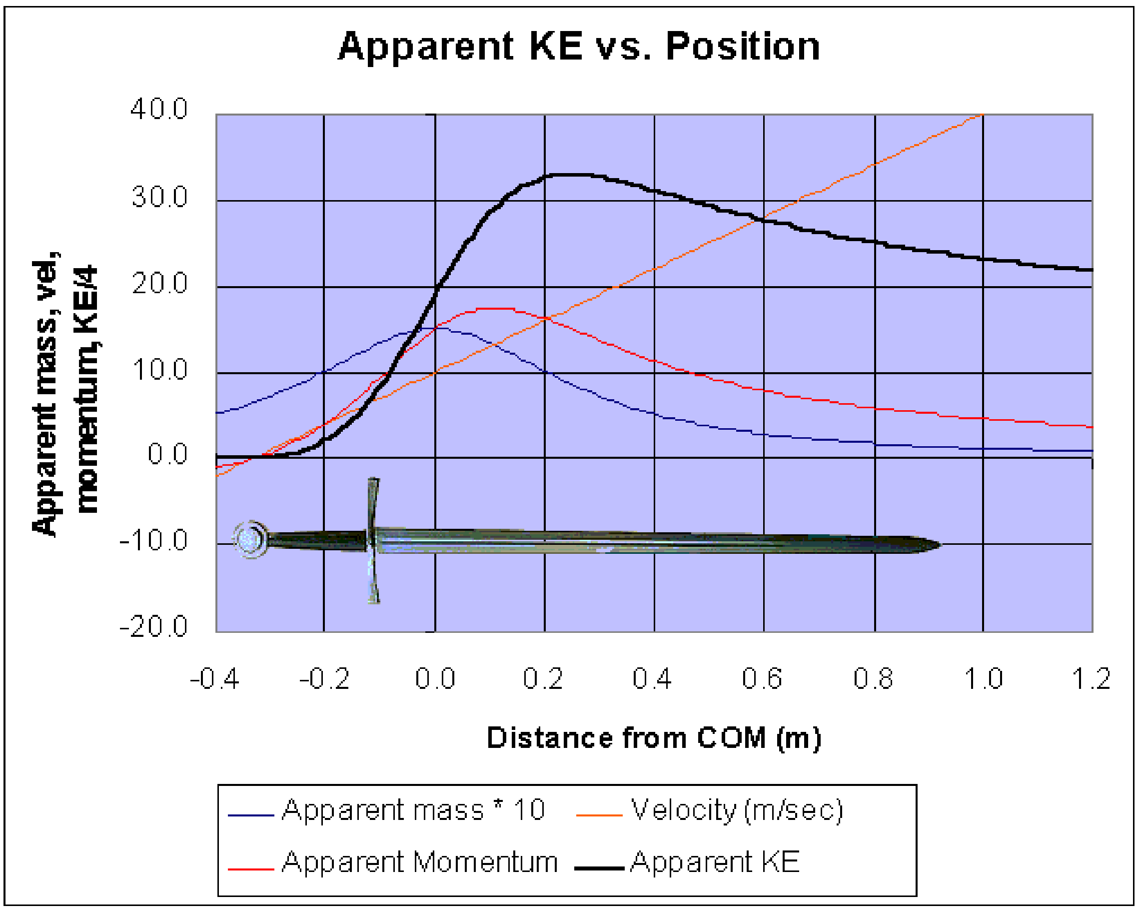


Here the blue curve represents the apparent mass, as calculated before, but scale up ten-fold to make it easier to see. Note that its curve always peaks at the swords center of mass, at coordinate $x = 0$. Also note that this curve is always symmetric about the center of mass. The velocity profile is the straight line that slopes upward toward the right. This line crosses $y = 0$ at the instantaneous center of rotation, and at $x = 0$ the line represents the value of the velocity of the swords center of mass, which in this example is 10 m/sec. The slope of the line, $slope = \frac{\Delta y}{\Delta x}$, is the angular velocity of the sword, and is in this case 30 radians/second.

The other curve is the apparent momentum, which is just the velocity line multiplied by the apparent mass curve. Note that it peaks a bit out from the sword's center of mass. This curve is only useful when trying to impart momentum to very heavy, solid objects, and we don't use this area in trying to damage an opponent. It might be useful if you were using your sword to bump a pot-bellied stove across a floor. We would like to have a handy formula for where this curve peaks, and by setting the derivative (which describes the slope of the curve) of the momentum function to zero, we can easily find this point. I'll skip taking you through the math and just give the result as $x_{PEAK\_MOMENTUM} = \dfrac{\sqrt{v_{COM}^2 + \omega^2 \cdot \dfrac{I_{COM}}{m}} - v_{COM}}{\omega}$. Not quite something you can solve in your head, and probably not something with an easy shortcut solution, either. The equation also doesn't give a result if the angular velocity of the sword is zero.

Sword Edge Kinetic Energy

A more interesting curve is one describing the apparent kinetic energy along the blade, where $KE_{APPARENT}(x) = \frac{1}{2} \cdot m_{APPARENT}(x) \cdot x^2$. This may seem like an odd formulation, since energy isn't a vector quantity, or anything like that. But to a target that gets hit by the blade, this would its blind calculation of the kinetic energy of the little piece of steel that slammed into it. To form this curve, we could multiply the apparent momentum curve by one-half the velocity curve, or multiply the apparent mass curve by one-half the square of the velocity curve. Either way we get the same function, which looks like this.



Here I divided the actual amount of apparent kinetic energy by four, to make it fit into the graph. You can see that the curve peaks a little further out on the blade than the momentum does. It also slopes up more steeply toward the left, and tapers off more gradually on the right. Unfortunately, the peak is also way inside the percussion point. This brings up another set of problems. The formula for the peak kinetic energy location is given by

$x_{PEAK\_KE} = \frac{\omega}{v_{COM}} \cdot \frac{I_{COM}}{m}$. This means that the peak kinetic energy location can be found by taking the rotational components, of both angular velocity and inertia, divided by the linear components, linear velocity and mass. However, we can use the percussion point formula to substitute for the moment of inertia to mass ratio. Since $Q \cdot x_Q = \frac{I_{COM}}{m}$,

$x_{PEAK\_KE} = \frac{\omega}{v_{COM}} \cdot Q \cdot x_Q$, where Q and x_Q are a set of percussion points, relative to each other. Ideally, we would want the impact point at x_{PEAK_KE}, coinciding with a desired percussion point location Q. If this is truly the case, then the formula becomes

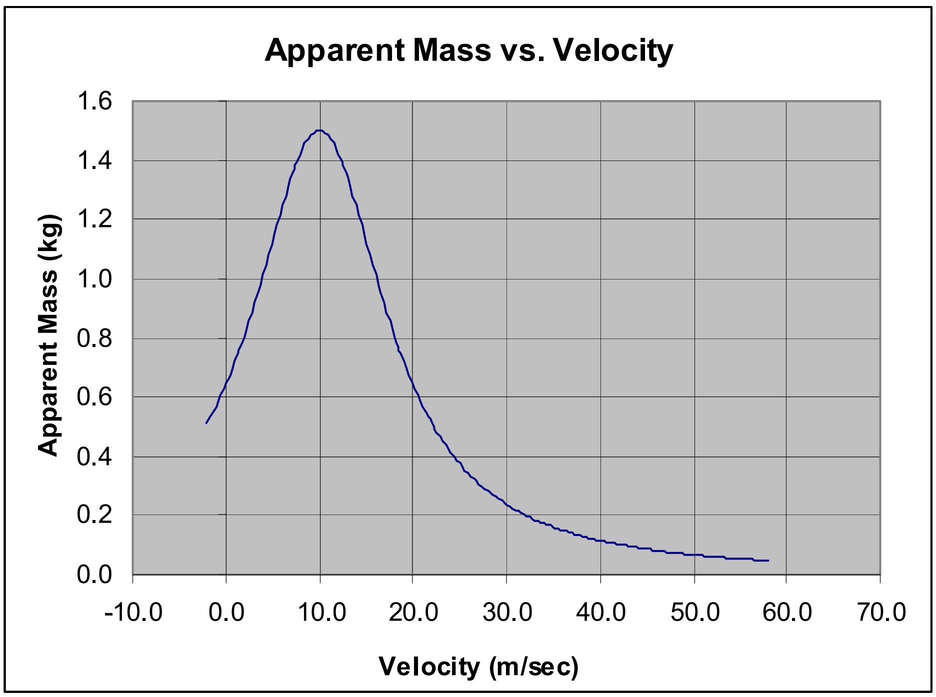
$Q = \frac{\varpi}{v_{COM}} \cdot Q \cdot x_Q$, where Q. cancels out, leaving us just $x_Q = \frac{v_{COM}}{\omega}$, or $v_{COM} = \omega \cdot x_Q$. But noting that the velocity of the center of mass is also found by multiplying the angular velocity ω by the distance from the center of mass to the instantaneous center of rotation, we find x_Q is also exactly same as the distance from IC to COM. **Therefore, the peak apparent kinetic energy in a swing occurs at the percussion point relative to the instantaneous center of rotation.** This also makes a great deal of sense, since an impact at the percussion point relative to the IC can completely stop all motion of the sword, bringing its total, final kinetic energy to zero.

However, this is also a bit depressing, since in most real swings the IC isn't all that near the hand, so the strikes that carry the maximum kinetic energy will not correspond to strikes that impart no hand shock. But this only spurs us on. Does the part of the blade containing the maximum kinetic energy actually do the most damage? What if damage follows some other function of apparent mass and edge velocity, instead of $\frac{1}{2}mv^2$? Since points located further out from the center of mass are traveling faster, might they deliver their kinetic energy in a more rapid pulse? Since kinetic energy is measured in Joules, and power in Watts, this concept would give us a function roughly following a function of $P = \frac{k}{2} \cdot m \cdot v^3$. This curve doesn't have a peak along the blade, it slopes upward continuously, steeply as approach the point of peak kinetic energy, the flattening out into a slight upward slope. We'll examine this in detail when we start modeling collisions.

Another important point to bring up is that the formula $x_{PEAK\_KE} = \frac{\omega}{v_{COM}} \cdot \frac{I_{COM}}{m}$ tells us that two basic ratios determine peak of the kinetic energy curve. If we add a pommel, the sword's center of mass shifts back toward the instantaneous center of rotation, meaning the ratio $\frac{\omega}{v_{COM}}$

increases, and more so than $\frac{I_{COM}}{m}$, so the peak point moves further out on the blade. The pommel not only shifts the percussion point toward the tip, it also shifts the peak in blade energy toward the tip, but by completely different means.

Since we already know how the apparent mass varies with distance from COM, and during a swing the edge velocity is just a linear function of distance from the COM, then it follows that we also know the curve of apparent mass versus edge velocity. Such a function looks like our previous function of apparent mass versus distance, but the units for distance have changed into velocity. This means that during a given swing, our choice of impact point boils down to deciding which point on this mass versus velocity graph will inflict the most damage. To figure that out, we need to investigate collisions between the sword and a target.



Chapter 9
Hands-Free Impacts

Let's examine the behavior of the sword as it impacts a particle of mass m_{TARGET}. We will use the adjusted mass of the sword at a point along the edge, and its corresponding apparent momentum and kinetic energy. Considering the momentum of the sword and target, which form the system under study, we can calculate the total momentum of the system, which must be conserved. If the target is initially stationary, then the initial momentum of the sword must equal the sum of the final momentums of both sword and target. This momentum is given by the formula

$m_{SWORD\_APPARENT} * v_{EDGE\_INITIAL} = m_{SWORD\_APPARENT} \cdot v_{EDGE\_FINAL} + m_{TARGET} \cdot v_{TARGET\_FINAL}$,

where the term $m_{SWORD\_APPARENT}$ is the adjusted mass that we've been calculating. It is this simplification that keeps this study from becoming involved in the complex rotational dynamics of the sword. Keep in mind that it certainly isn't the actual mass of the sword. To further simplify the variable names, I'll switch to m_{SA} for the inertia of the sword, v_{SI} and v_{SF} for the initial and final velocities of the sword's edge, with m_T and v_{TF} for the target's mass and final velocity. This shorthand reduces the above equation to $m_{SA} \cdot v_{SI} = m_{SA} \cdot v_{SF} + m_T \cdot v_{TF}$.

Since the target is initially unmoving, the approach velocity in this case is just the sword's edge velocity v_{SI}. The velocity of separation of the sword and target is given by $v_{TF} - v_{SF}$. The ratio of the separation versus the approach velocity defines the coefficient of restitution of the impact, abbreviated e, which can vary from 0 for perfectly inelastic collisions, to 1 for perfectly elastic collisions. A perfectly inelastic collision will leave the sword and target traveling with the same final velocity, and this type of collision would characterize a cut on a soft target. A perfectly elastic collision might occur with an easy swing against a steel target, where the sword bounces off armor without making a dent. For a given value of the coefficient of restitution, we have two equations with two unknowns, making it trivial to solve for the final velocities of both sword and target. For example, since $e = \frac{v_{TF} - v_{SF}}{v_{SI}}$, then $v_{TF} - v_{SF} = e \cdot v_{SI}$, but we also have the formula for the conservation of momentum, so we get $v_{TF} = \frac{v_{SI} \cdot (e+1)}{1 + \frac{m_T}{m_{SA}}}$, and thus

$v_{SF} = v_{TF} - e \cdot v_{SI}$. The maximum possible target velocity would occur if a perfectly elastic collision where the mass of the target was negligible compared to the mass of the sword, making the denominator of this equation 1. This would give a final target velocity equal to twice the sword's initial velocity, where the approach velocity has become the departure velocity, but the sword's velocity is unchanged. For a perfectly inelastic collision, the target velocity becomes some fraction of the sword's initial velocity, as determined by dividing the sword's initial velocity by one plus the ratio of the relative masses. But again, keep in mind that we're using the adjusted mass of the sword.

If we allow the target to have some initial velocity, which will be negative if the target is approaching the blade, like a baseball, then the equation for final target velocity becomes

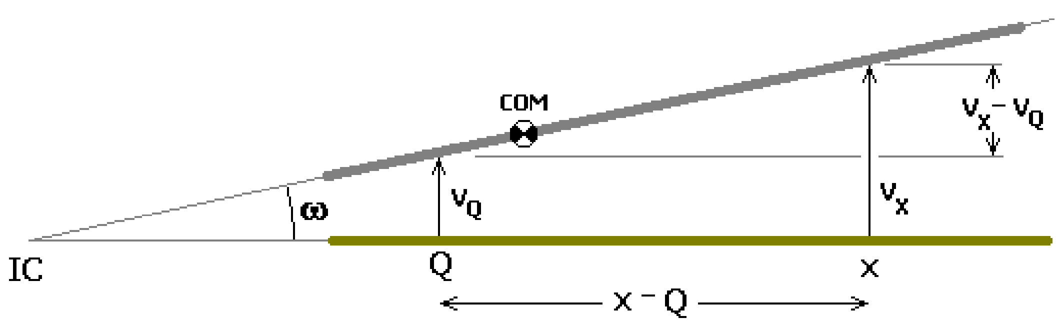
$$v_{TF} = \frac{v_{SI} \cdot (e+1) + \left(\frac{m_T}{m_{SA}} - e \right) \cdot v_{TI}}{1 + \frac{m_T}{m_{SA}}}$$

, and $v_{SF} = v_{TF} - e \cdot (v_{SI} - v_{TI})$. This makes it a bit simpler to calculate the results of hits on moving targets.

Post-Impact Linear and Angular Velocities of the Sword

We can now calculate the final velocity of the point on the edge that impacts the target. However, we also know that the percussion point, relative to this impact point, undergoes absolutely no change in velocity. Much of our discussion of percussion points has centered on this very fact. All this means that we can calculate the post impact velocity of two points on the sword, which means we can also calculate the sword's final linear and angular velocities.

Given an impact point on the edge located at distance x from the sword's center of mass, the corresponding percussion point location is given by $Q_X = \frac{I_{COM}}{m_S \cdot x}$. However, this is the distance is given as the distance from the center of mass, on the opposite side of the center of mass from the impact point. So the actual location, with a coordinate system centered on the sword's center of mass, with the impact point being positive, will be given by $Q_X = \frac{-I_{COM}}{m_S \cdot x}$. The initial velocity of this point is $v_Q = v_{COM\_INITIAL} + Q \cdot \omega_I$, where $v_{COM\_INITIAL}$ is the initial linear velocity of the sword, and ω_I is the initial angular velocity of the sword.



In the figure, the instantaneous center of rotation, angular velocity, and linear velocities could be either pre or post impact. The math is the same in either case, although in some post-impact scenarios, the edge velocity at x will be less than the velocity point Q, in which case the sword is rotating around the target, instead of your hands. This happens when you swing at a lamppost.

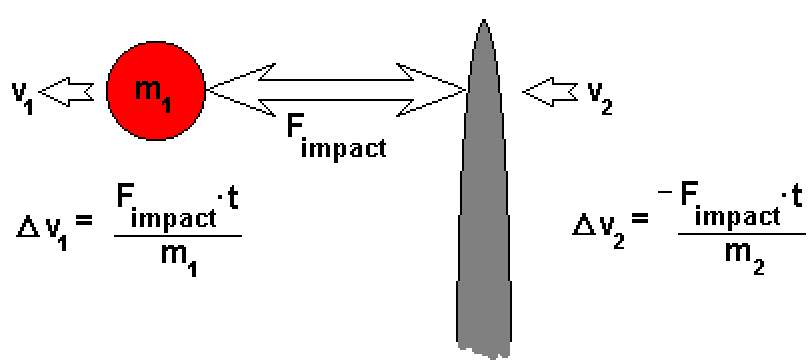
From the figure, and simple geometry, we can get the formulas for the post impact conditions.

$$\omega = \frac{v_X - v_Q}{x - Q}$$
$$v_{COM} = v_X - \omega \cdot x$$

Once we know the final angular and linear velocities, we can illustrate how much kinetic energy remains in the sword, how much in the target, and how much was dissipated in the form of violence done to the target. We can also show that the total momentum of the collision is conserved, based on the true mass of the sword, its linear and angular velocities, and the targets mass and final velocity.

Linear Impulse

The equations given above work quite well, but are a bit more cumbersome than necessary. One simplification comes from an understanding of linear impulse, which is merely the change in linear momentum of a system. If you apply some force F to a system, which possesses a constant inertia given by m, the resulting acceleration is of course just a. We know the equation for the final velocity of a system, when put under some acceleration a, which is simply $v_{FINAL} = v_{INITIAL} + a \cdot t$. Or in simpler terms, the change in velocity, from initial to final, is given as $\Delta v = a \cdot t$. The force causing this change is also being applied for the time t, so if we multiply both sides of Newton's equation $F = m \cdot a$ by time t, you get $F \cdot t = m \cdot a \cdot t$. But we've already shown that $a \cdot t = \Delta v$, so $F \cdot t = m \cdot \Delta v$. The right hand side of the equation is merely the change in momentum of the system. Even if the force varies, which it will do throughout the cut, we still end up with the equation $\int F \cdot dt = m\Delta v$. This just means that we have something called impulse, which is just the change in momentum of a system. Since force is measured in Newtons, and time in seconds, the change in momentum is given in Netwon seconds. The units work out, as a Newton is a $kg \cdot m / \sec^2$, which is just mass times acceleration, so a Netwon second is $(kg \cdot m / \sec^2) \cdot \sec$, or $kg \cdot m / \sec$, and an inertia times a velocity is also expressed in units of $kg \cdot m / \sec$. The more common unit for an impulse is, however, the Newton-second, so that's the unit that will be used throughout the rest of this analysis. Interestingly, solid rocket motors are rated in Newton-seconds, so our sword research has evolved somewhat into rocket science. Rocketry and satelites are also one of the few fields where working with mass moment of inertia is very, very important for trajectory and orientation control, since the rocket's mass moment of inertia is dominated by the mass of the remaining fuel, which is continously varying.



So if we use the previous equation for the final velocity of the target, which is just

$$v_{TF} = \frac{v_{SI} \cdot (e+1) + \left(\frac{m_T}{m_{SA}} - e \right) \cdot v_{TI}}{1 + \frac{m_T}{m_{SA}}}$$

, we can calculate the linear impulse applied to the target as simply $impulse = m_T \cdot (v_{TF} - v_{TI})$. Here I'm taking the impulse to be positive if the target velocity increases, and the sword edge velocity decreases. The same impulse is applied, in equal amount and in the opposite direction, to the blade edge, so $impulse = m_{SA} \cdot (v_{SI} - v_{SF})$. So

$v_{SF} = v_{SI} + \frac{m_T}{m_{SA}} \cdot (v_{TI} - v_{TF})$. Again, this just means that the ratio in the change in velocities is always inversly proportional to the ratio of the masses, or in this case the inertias. So we can easily find the final velocity of the sword edge. To find the final velocity of the sword's center of mass is just as easy.

$$v_{COM\_FINAL} = v_{COM\_INITIAL} - \frac{impulse}{m_{SWORD}}$$

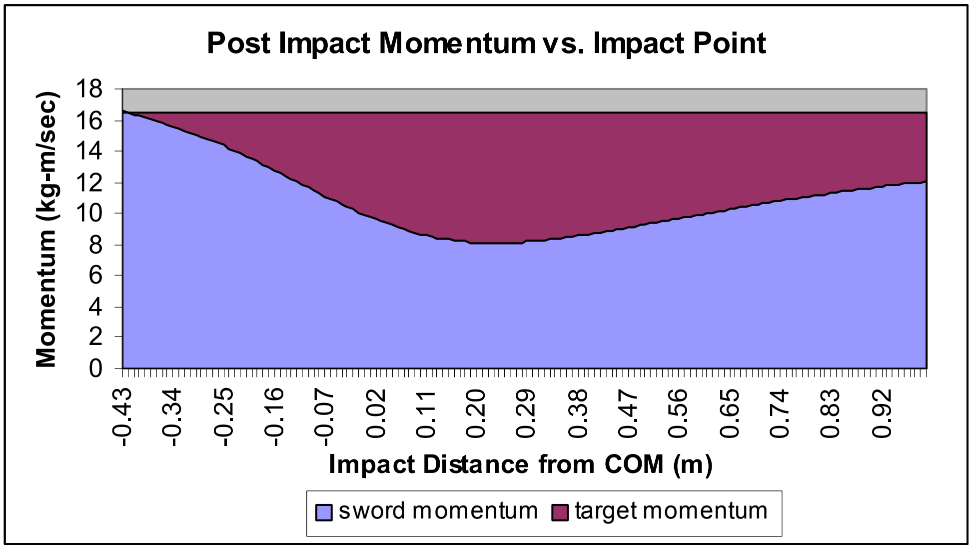
In the above equation, m_{SWORD} is the sword's true mass, and the impulse is exactly the same as we applied to the target, which was positive. Now we know the true velocities of two points on the sword, the center of mass and the edge that impacted the target. The angular velocity of the sword is now trivial to calculate, as it's the difference in velocities divided by the distance between them.

$$\omega = \frac{v_{EDGE} - v_{COM}}{x}$$

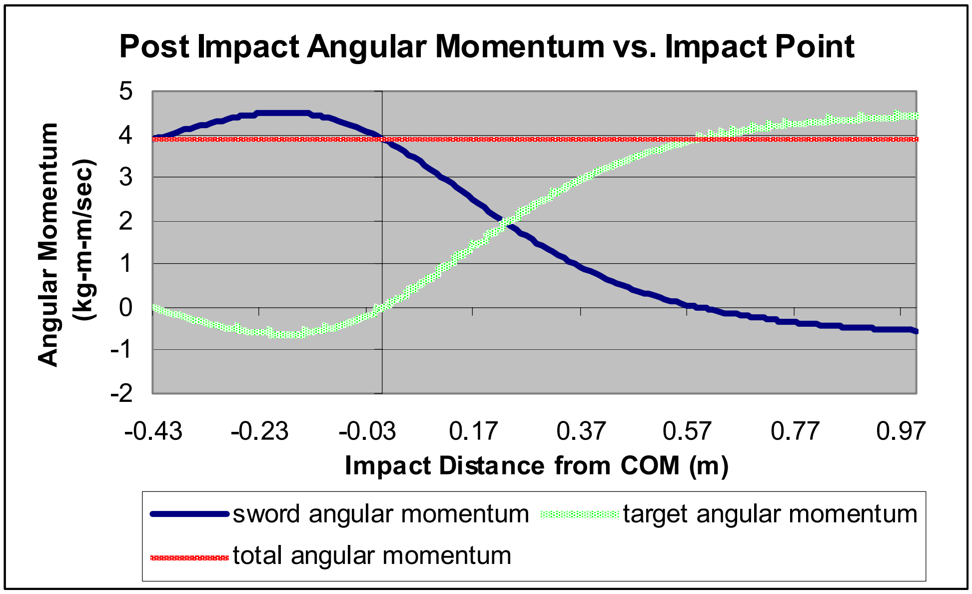
In this equation, x is just the distance from the sword's center of mass to the impact point, which we already knew, in order to calculate the apparent mass, or inertia, of the sword's edge at that impact point. So we can now easily build spreadsheets and graph impact results, which reveals some very interesting behaviors.

Graphical Analysis of Impacts

Here is one such chart, and the sword's momentum plus the target's momentum sum to give the 16.5 kg-m/sec, which remains constant regardless of impact point.



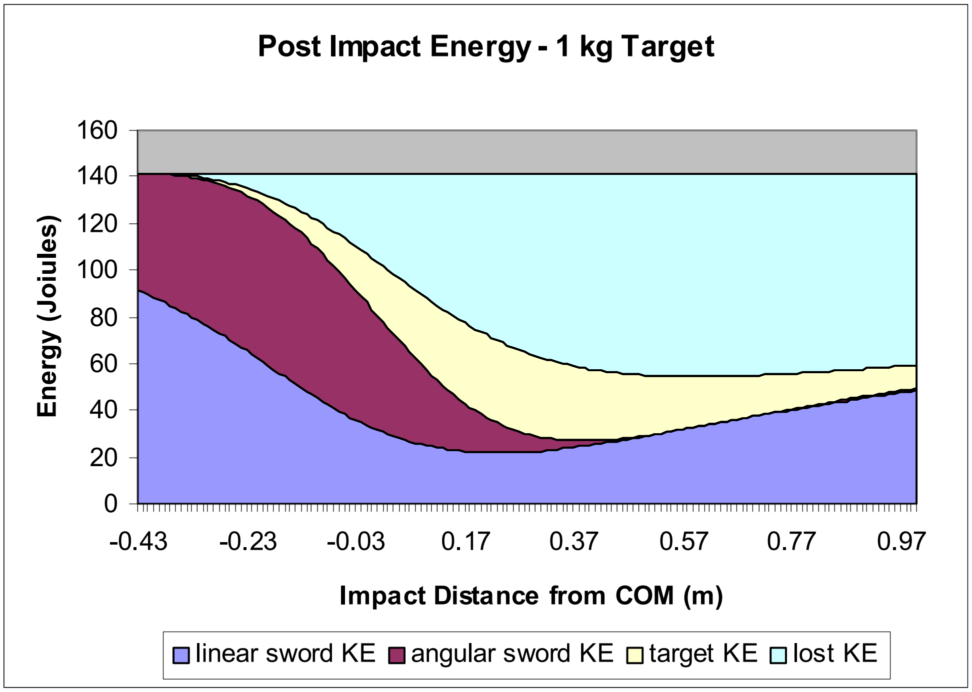
This confirms that the apparent inertia model satisfies the conservation of momentum, which all impacts must do, lest they disobey the laws of physics.



This chart shows that the sum of the sword's angular momentum and the target's angular momentum remains constant, regardless of impact point. All impacts must also satisfy the conservation of angular momentum, and the apparent inertia model does this, too.

We now calculate the energies. $KE_{LINEAR} = \frac{1}{2} \cdot m_{SWORD} \cdot v_{COM}^2$, $KE_{ANGULAR} = \frac{1}{2} \cdot I_{COM} \cdot \omega^2$.

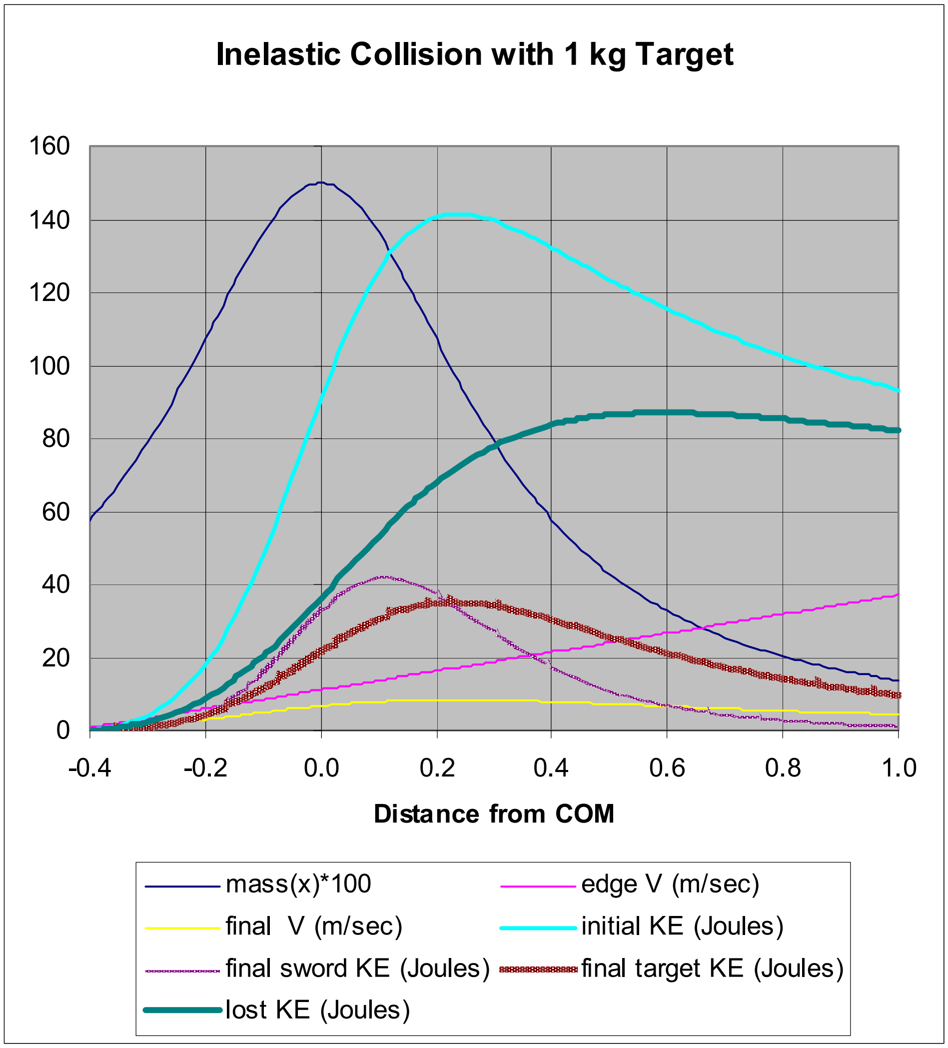
The target's kinetic energy is also just $KE_{TARGET} = \frac{1}{2} \cdot m_{TARGET} \cdot v_{TARGET}^2$.



In this graph, you can see that for hits toward the hilt, not much is happening. Where the top area is thickest is the point of maximum damage. It's referred to as lost kinetic energy because it's no longer in the form of measurable kinetic energy. It's been transformed into mechanical deformations, tissue damage, or blade damage. The area underneath represents how well the impact propelled the target, which is maximized a bit inward of the maximum damage point. If you want to hit a baseball, this is the part of the graph you'd want to optimize. However, this graph is for an inelastic collision, which would be more like hitting a melon than a ball. That's why the post impact target kinetic energy is so small. The next portion of the graph is the angular energy of the sword, considered as rotation remaining around the sword's center of mass. For impacts very close to the hilt, the sword is still rotating in the forward direction. In these cases, you're actually continuing to rotate *around* the target. The bottom portion of the graph is the linear kinetic energy of the sword, which is just due to the motion of its center of mass. It remains large for impacts way inside the center of mass, because you're actually hitting with the pommel or hilt, and the impact isn't bringing this motion to a halt.

Applying the Collision Results

In an inelastic collision, the sword and target end up traveling with the same final velocity. What we are doing with the impact, however, is trying to maximize the loss in kinetic energy of the sword and target system. The lost kinetic energy is being used to deliver damage, while retained kinetic energy just moves the target around, or remains in the sword.
Here the thin line starting near 60 on the left, at x=-0.4, traces the apparent mass of the sword along its length. As you can see, its peak is centered on the sword's center of mass. Next to this peak is the apparent initial kinetic energy in the sword, which peaks well inside the percussion point relative to the hand. Peaking directly beneath it is the curve of the target's final kinetic energy. This represents energy of motion, and is not energy delivered as damage. In between these last two is the curve of the lost kinetic energy, which does represent damage.



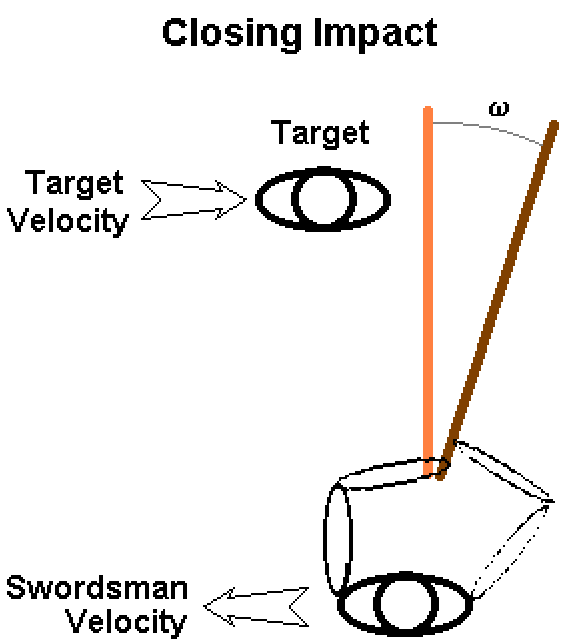
Note that its peak occurs well past that of the kinetic energy of either sword or target. That means that for this particular target, the maximum damage is being done further out. This particular sword has a percussion point, relative to the hand, located 0.5 meters from the center of mass. This is also quite close to the peak in the curve representing lost kinetic energy.

Now let's look at lots of examples of impacts, and observe behaviors.

Variations in Closing Velocity

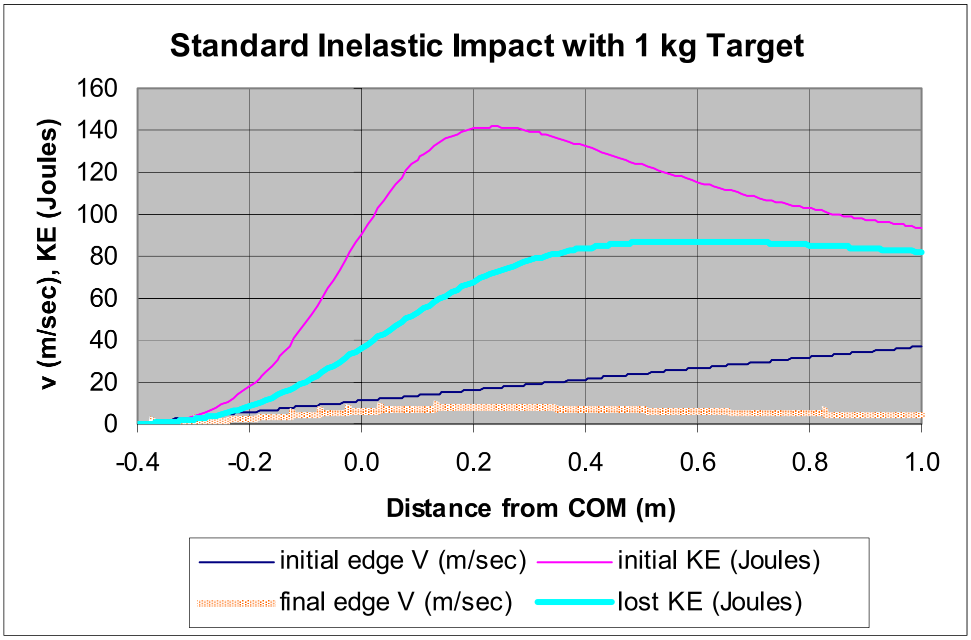
We've been ignoring the target's own initial velocity, for a couple of reasons. Often, the target is fixed, or moving slowly enough that it can be ignored. If you strike at someone's lower leg, which is firmly planted, the initial leg velocity is zero. Also, we can easily just add the target velocity to our pre-impact linear sword velocity, v_{COM}, and get the same result. However, we ignore the component of target velocity that moves back and forth along the edge, and only use the target velocity perpendicular to the edge.

So if you are striking from right to left, and your target is moving from your left to your right, the closing velocity increases. If you are also traversing left, your closing velocity goes up yet more. This serves only to increase the effective linear sword velocity v_{COM}. But if you are traversing to your right, and your target is moving to your left, the closing velocity is decreased. Contrary to the commonly held assumption that there is a single best point on the blade to strike with, you actually need to vary the blade impact location based on these changes in closing velocity. But note how closing velocity is being determined. It is unrelated to the act of closing with your opponent, which means closing the distance.

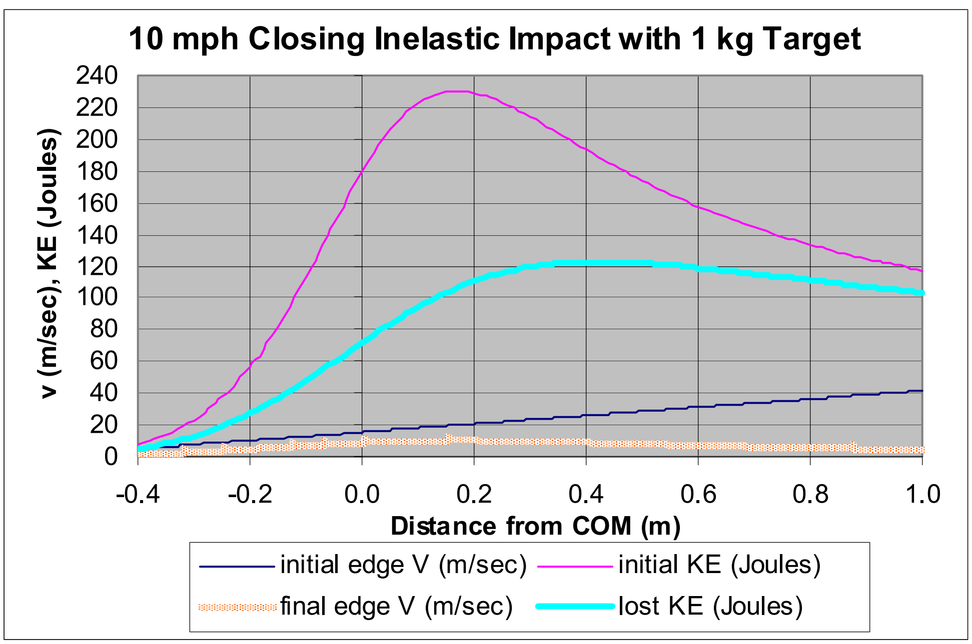


Here, the closing velocity of the target and swordsman can be added to the sword's center of mass velocity, v_{COM}. You might somehow think that this also affects the angular velocity of the sword, but this is not so. The angular velocity can be measured in radians per second, or even revolutions per minute (rpm). If your car engine is ticking over at 2500 rpm, then that's what it is, regardless of your speed. Similarly, an artillery shell's rotation rate is fixed as it exits the muzzle, and isn't linked to the velocity of the shell throughout its flight. The angular velocity of the sword is entirely independent of variations in closing velocity, or the linear velocity of the swordsman. However, this closing velocity is added directly to the center of mass velocity, and by previously derived math, this means that the location of the instantaneous center of rotation is directly affected. In a closing strike, the calculated IC is moving way inside, far from the sword. Imagine striking a mailbox from a car. As your sword passes through the last 1 degree of rotation prior to impact, your car has traveled a great distance along the road. If you plot the two lines made from the tip to hilt of your sword, they intersect somewhere across the other side of the road, past the tree line. Your swing looks the same to you, but not at all the same to your target. The point of maximum blade kinetic energy corresponds to the percussion point relative to the instantaneous center of rotation. During a closing impact, the distance from the sword's center of mass to the IC gets larger, so the corresponding distance between the IC's percussion point and the COM must be getting smaller.

Another way to look at this is with another graph of an inelastic impact with a 1 kg target.



If we add in a 10 mph closing velocity, we get the following changes.

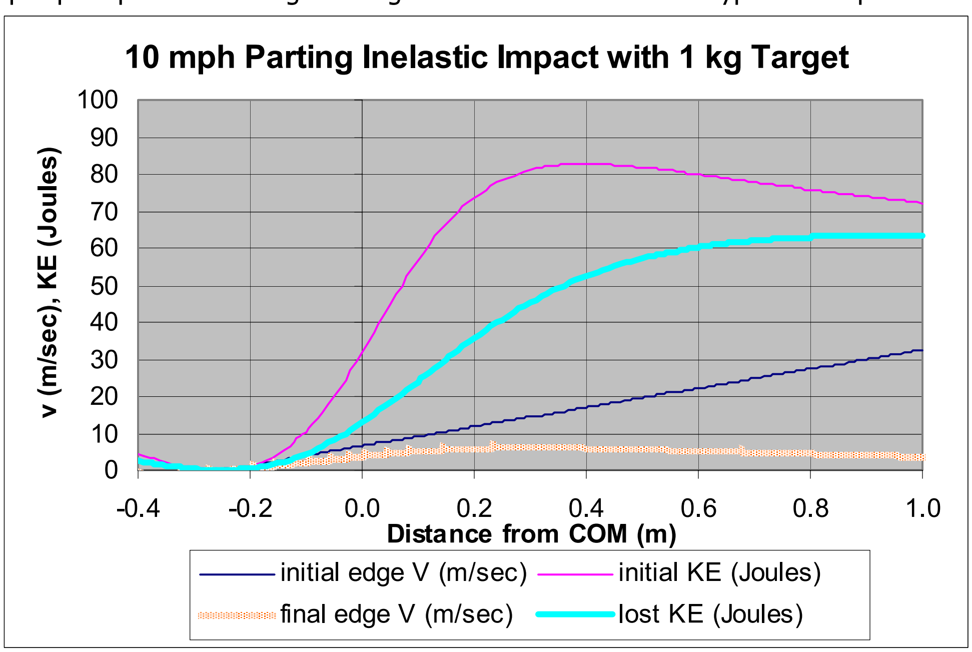


Comparing the two charts gives us quite a bit to work with. Looking at the formula for the location of peak kinetic energy in the blade, $x_{PEAK\_KE} = \frac{\omega}{v_{COM}} \cdot \frac{I_{COM}}{m}$, increasing v_{COM} by closing on the target decreases the location for peak kinetic energy. The kinetic energy dissipated in the impact also peaks closer to the blade. Also noteworthy is that both curves slope downwards more steeply, as the impact approaches the tip. Basically, by sliding the line representing the linear velocity upward, the apparent inertia near the hilt has been given a bigger multiplier. Since the apparent inertia near the hilt is so much larger near the hilt, the upward shift in the edge velocity line brings the peaks back, and causes them to be much less favorable as you approach the tip.

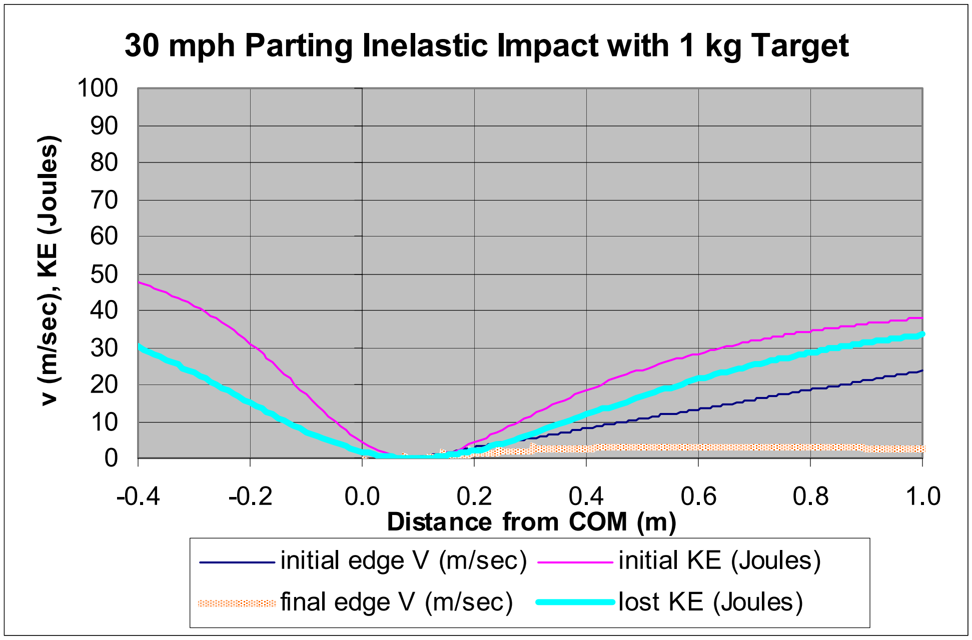
Another noteworthy feature is the large increase in both sword edge kinetic energy, and delivered energy, all just by increasing the linear velocity of the impact. We've gone from delivering 87 Joules at 0.60 meters up to 122 Joules at 0.42 meters from the COM, an increase of 40%. The peak blade edge energy has gone up from 141 Joules at 0.24 meters to 230 Joules at 0.17 meters from the COM, a 63% increase in energy. But at the tip, the closing impact can only deliver 103 Joules, which is only 84% of the energy delivered at the curve's peak. In the standard impact, the tip could deliver 94% of the energy at the peak. So even though the tip itself does more absolute damage during the closing impact, there is much more damage that can be done by striking further back on the blade.

A parting impact is much weaker, and the peaks of the curves have shifted toward the tip.
In our 10 mph parting shot, the peak kinetic energy is 0.40 meters from the center of mass. The peak in dissipated kinetic energy is now actually at the tip. But even this peak, at 63.5 Joules, is significantly lower that the 87 Joules delivered in the standard impact, just 73% as much. If we struck with the standard location of maximum damage, where the 87 Joules was delivered, we'd only deliver 60 Joules, or 69% as much.

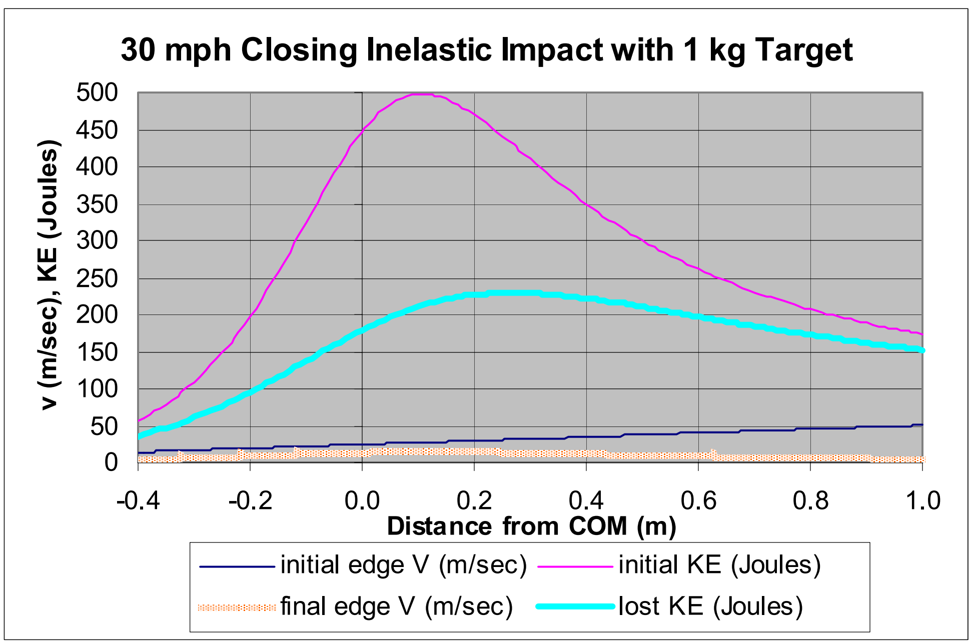
By the way, the reason the left edge of the chart shows increasing energy is simple. This upslope represents hitting the target with the back end of a hypothetical pommel extension.



If we were on a galloping horse and tried to swing down and back at a target, you can see from the chart that we'd better use the tip. But even then, we'll deliver only 33 Joules. The zero point on the graph, located a bit forward of the center of mass, isn't moving at all, relative to the ground.

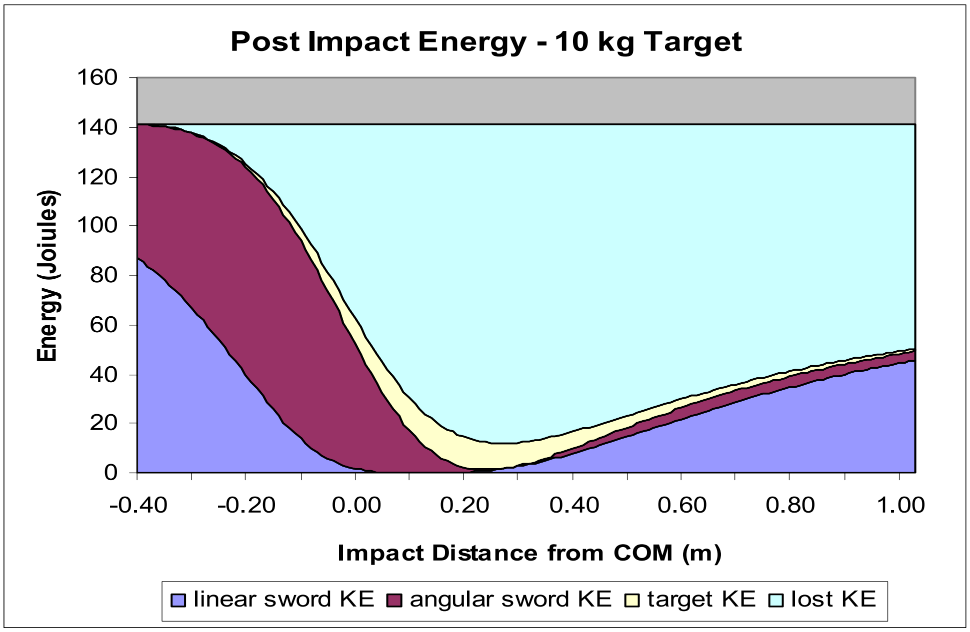


The high-speed closing curve looks nothing like the parting curve. It can deliver 229 Joules, at a point 0.26 meters from the center of mass. That's about seven times more energy that the tip delivered during a parting shot. But the peaks in the curves are much closer to the center of mass.

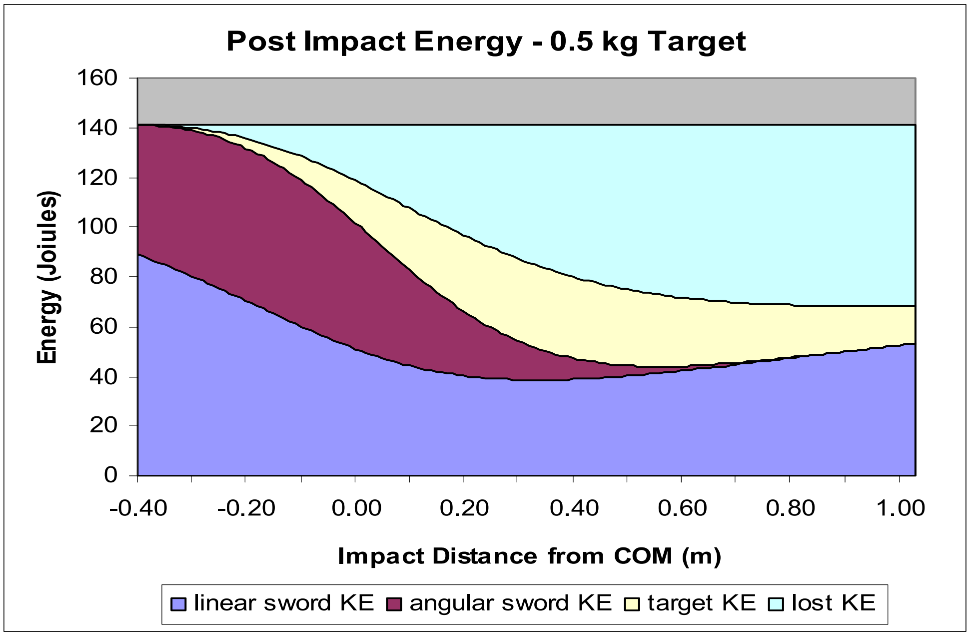


Mass of the Target

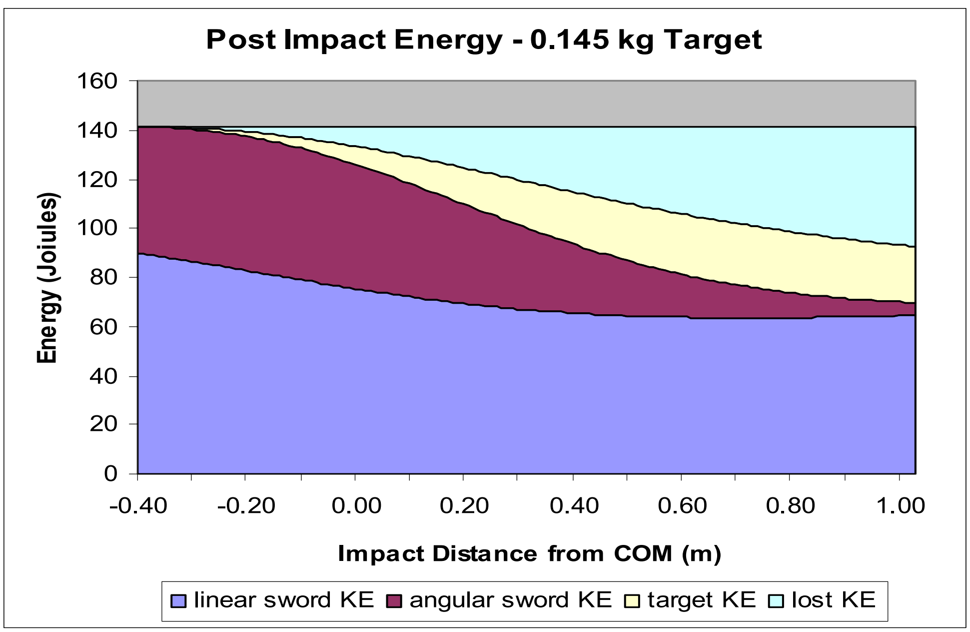
Target mass plays a huge role in the outcome of the collisions. The graphs of energy loss look quite different with even fairly small changes in target mass. A 10 kg target has peaks much closer to the sword's center of mass, while 0.1 kg targets keep sloping up as you go off the end of the sword. For one thing, this says strike extremely light targets, like small birds, with the tip. Strike heavy targets, like tree trunks, further back on the blade. This behavior is significant, complex, and directly affects a swordsman. Just because you know the mass of your opponent doesn't mean you know how much of that mass is actually involved in the impact. This is something to be empirically investigated. The foundations of that research are given here.



Here is a cut against a very heavy target, which would be representative a large piece of firewood. You can see that the best option is to hit pretty much with the part of the blade that brings its total energy to zero. This is far back on the blade, and will generate tremendous hand shock, at least on most swords. This is one reason that we cut wood with an axe, instead of a sword. It also shows that cutting tests against heavy, solid targets are very misleading. Optimizing a sword for this kind of cut is going to product something other than a proper sword. It would also teach you to strike very far back from the tip, even though that generates a great deal of shock, since against this target you can see the "improvements" with your own eyes.



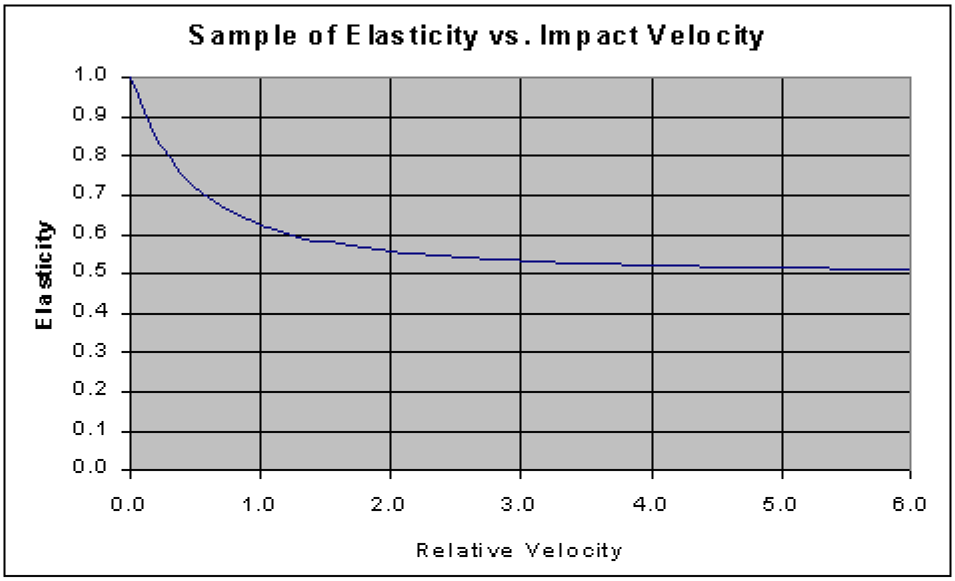
Here is a strike against a more reasonable target, which weighs a bit more than a pound. Here, the whole tip section is inflicting good damage, and the impact point is not very important.



Here is a chart of a strike against a light target, specifically an inelastic collision with a baseball. The elasticity is still calculated as zero, on the assumption that your blade is very sharp. The sword is only able to dump about half of its total kinetic energy, and a good percentage of that is still going into the kinetic energy of the target, instead of cutting. The lighter the target, the worse this problem becomes. But every sword has a roll over point, where targets of sufficient lightness are best cut with the tip. Lighter swords reach this point while still cutting very light targets. Heavier swords retain their optimal impact point at the tip against correspondingly heavier targets. All these graphs show that the best part of the blade to strike with is an elusive area. It depends heavily on the mass of the target.

Elasticity of the Target

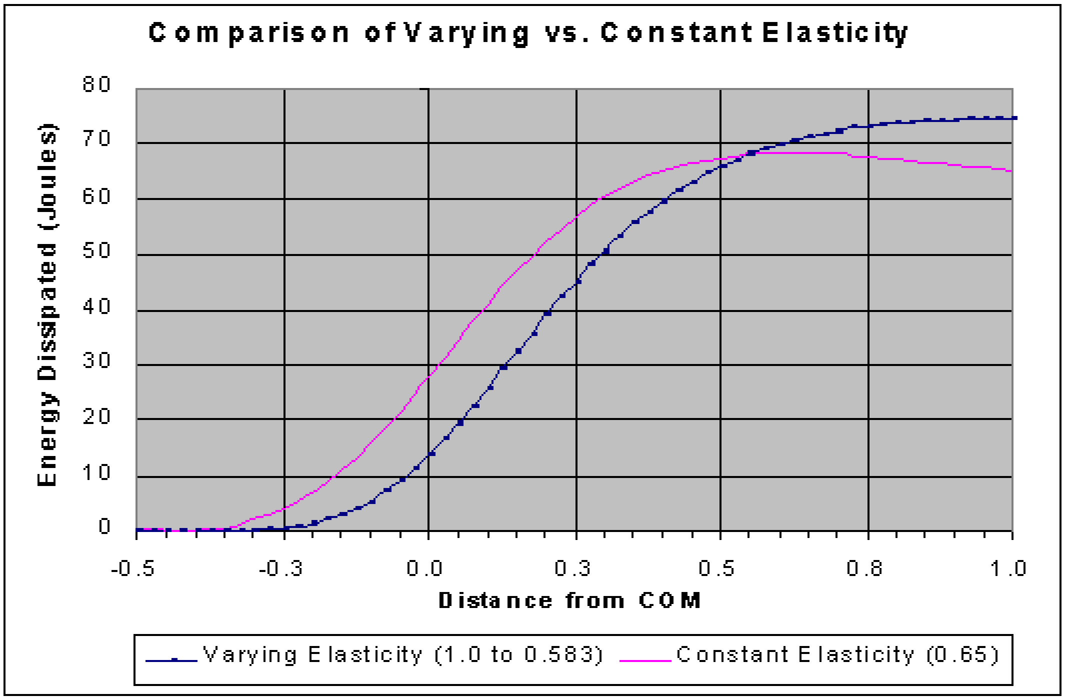
The preceding graphs have illustrated inelastic collisions, because an elastic collision can't cause mechanical damage. In an elastic collision, the initial kinetic energy is retained completely as kinetic energy, leaving no energy left over to cause mechanical deformation of armor, or even tissue trauma. However, light steel-on-steel collisions are considered perfectly elastic, or $e = 1.0$. As the impact velocity increases, the elasticity of a steel-on-steel collision drops, to somewhere near $e = 0.6$, or even lower. How this exactly occurs depends on a huge number of variables.



If the target material is soft steel, it will more easily deform. If it's spring steel, the impact will likely be more elastic. If it's a thin, poorly supported piece of plate, it's more likely to crease or dent. Similar arguments apply to leather armor and quilted gambesons, which at low velocities will just give, unless backed by something hard and heavy, like an anvil. Once you start cutting flesh you start picking up some problems with viscoelasticity. In a very slow impact, you push blood around, you stretch muscles, but you don't necessarily do any permanent damage, or even cause pain. You're just shoving meat around. So even though the blow dissipates energy into the target, nowhere is the energy density in the target high enough to damage anything. An example of this would be stepping into a blow before its power developes, and stiffling it. You can use your arm or body to absorb the energy of the blow at a slow rate, and suffer no damage.

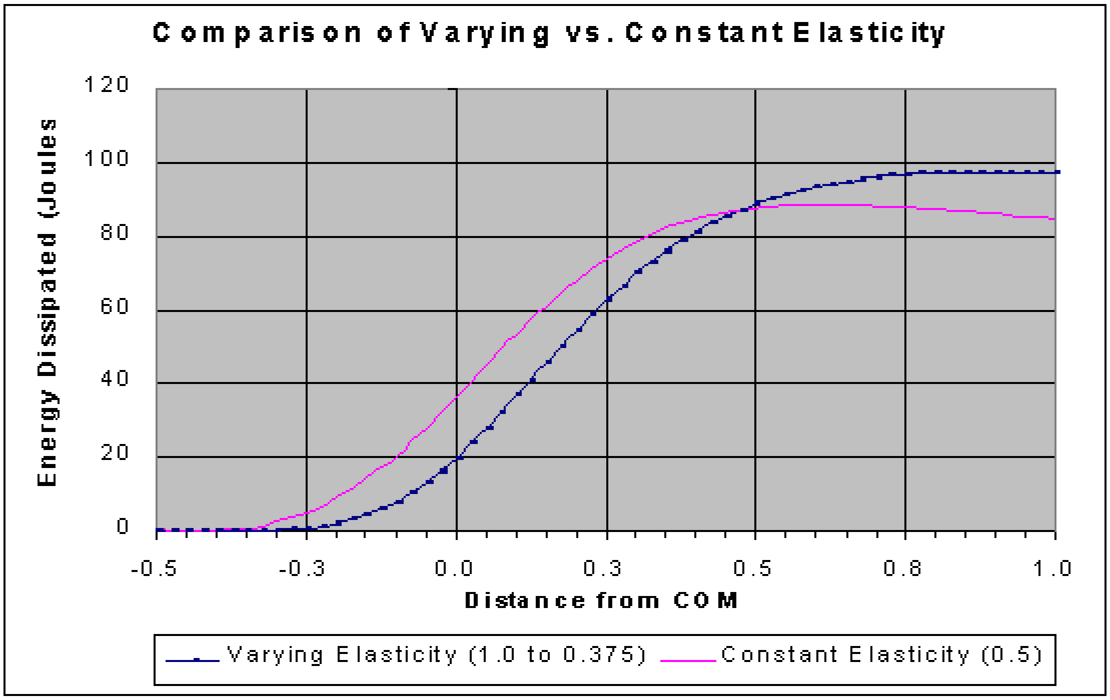
All of these subjects need to be examined experimentally, and the data recorded. Designing or understanding a weapon's fundamentals requires knowing the relevant properties of the target. If tasked to design a new rifle cartridge, you have to know if it's for hunting squirrel, elk, or armored vehicles. However, even at this stage we can safely say that the impacts elasticity drops with increasing impact velocity, and we've already examined many cases that have constant elasticity, which are easily analyzed. So let's look at some representative possibilities of decreases in elasticity as the impact approaches the tip, which is the same as observing the impact results as the closing velocity increases.

Let's compare an impact with constant elasticity with an impact that has decreasing elasticity as closing velocity increases.



In the above chart, we have two swings with identical parameters, except for the elasticity of the collision. One plot shows a constant elasticity of 0.65, which is moderately elastic. The other plot has elasticity that decreases with increasing velocity. The elasticity is 1.0 for zero closing velocity, and decreases to a value of 0.583 at the tip velocity, which in this case was 51 m/sec. The exact nature and magnitude of the change in elasticity is not the focus of the chart. The key point is that if elasticity decreases toward the tip, then the dissipated energy must increase, relative to the constant case, toward the tip, since the impact becomes more inelastic. Also important is that the lower closing velocities found near the hilt result in greatly decreased dissipated energy near the swing's instantaneous center of rotation. No matter what formula you use to model the decrease in elasticity with increasing velocity, the gross affect is somewhat similar. Compared to what you would otherwise have, the peak in dissipated energy shifts outwards from the swing's center.

Here's another chart with a different formula for the decrease in elasticity.



This chart uses a different formula for the decrease in elasticity, and the reference curve has an elasticity of 0.5. The elasticity of the reference was decreased to keep the curves somewhat similar. The average elasticity in the curve of varying elasticity is 0.653, which is similar to the constant elasticity in the first graph. If we compared this graph's reference curve of dissipated energy, it's at 84 Joules at the sword's tip. If we used an elasticity of 0.65 the constant elasticity curve would only be 65 Joules at the tip. Having impact elasticity decrease with velocity can make a large difference in dissipated energy across the blade, wiping out your inside performance but increasing your tipward performance. If studies on representative real world target materials, such as meat, leather, and steel armor, shows that the decrease in impact inelasticity is anywhere near as large as in these graphs, then cutting near the tip will show itself as by far the best option for inflicting damage.

Chapter 10
Impacts with Hands

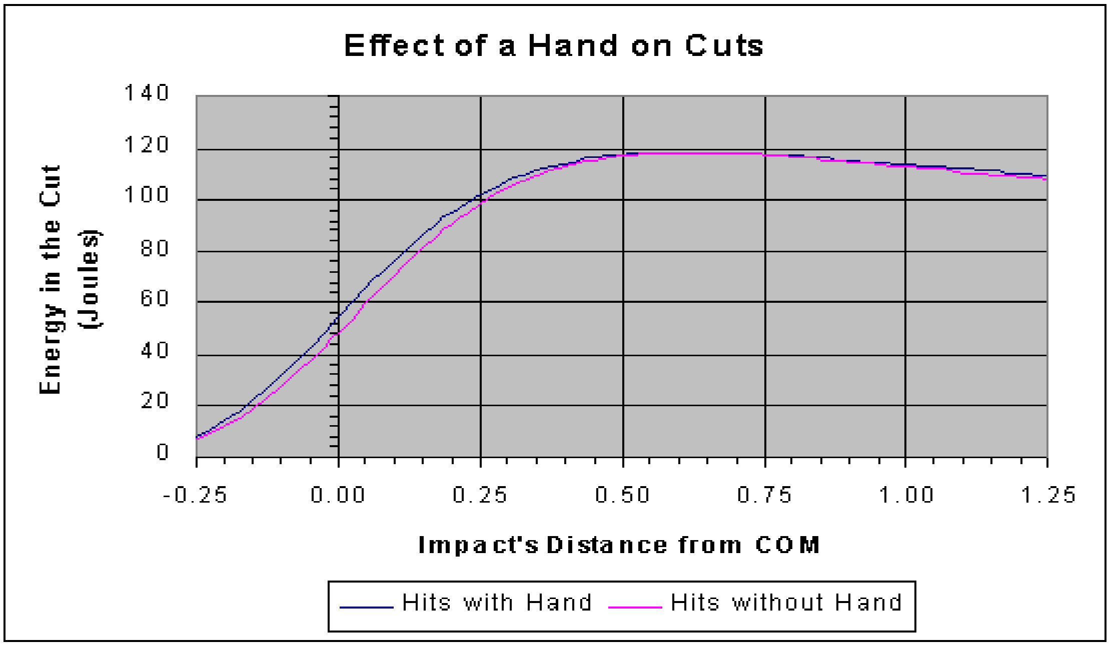
We don't swing the sword without holding onto the handle, so we definitely need to know the affect of our hands, whether one or both. The question involves how to model the hand. If you regard them as solid objects that are physically and solidly attached to the handle, you could add them in as simple masses, just like the pommel. Then you would generate a new description of the sword, which includes having one or two extra "pommels" on the handle. However, a hand doesn't really resemble a piece of steel that's welded to the hilt. If you look closely at your grip on the handle, you'll notice a bit of slop, or looseness, as your skin and muscles compress. There are a couple of simple ways to model this type of contact.

We could say that the force between the hand and handle builds up linearly as they compress together. This would turn the hand into a spring mass system, which should be fairly easy to analyze. However, a pure spring mass system doesn't dissipate energy, it conserves it. That wouldn't leave any energy left to cause pain, or break your wrist, if it comes to that. So we would at least have to treat the system as a damped spring mass system. Then we would expect to feel several oscillations, with the handle kicking alternately into our palm, then into our fingers. But generally we don't feel that kind of behavior in a hard impact. We feel a single, solid hit, followed by some high frequency blade vibrations. So I'll skip this model till we have data that might point to its validity.

If we simply treat the behavior of the hand and the handle as an impact between the two, we might get something closer to the truth. When the blade sharply impacts a target, it undergoes a sharp change in linear and angular velocity, which can result in a rapid change in the velocity of the handle if the impact was far from the handle's percussion point. The hand and handle are quite suddenly travelling at different velocities, which of course results in a collision. This collision changes the linear and final velocities of the sword. If the cut is not complete, we can then recalculate through more of the cut, which again changes the handle velocity, which causes another collision, and on and on. The model is quite crude, but might point us in the right direction.

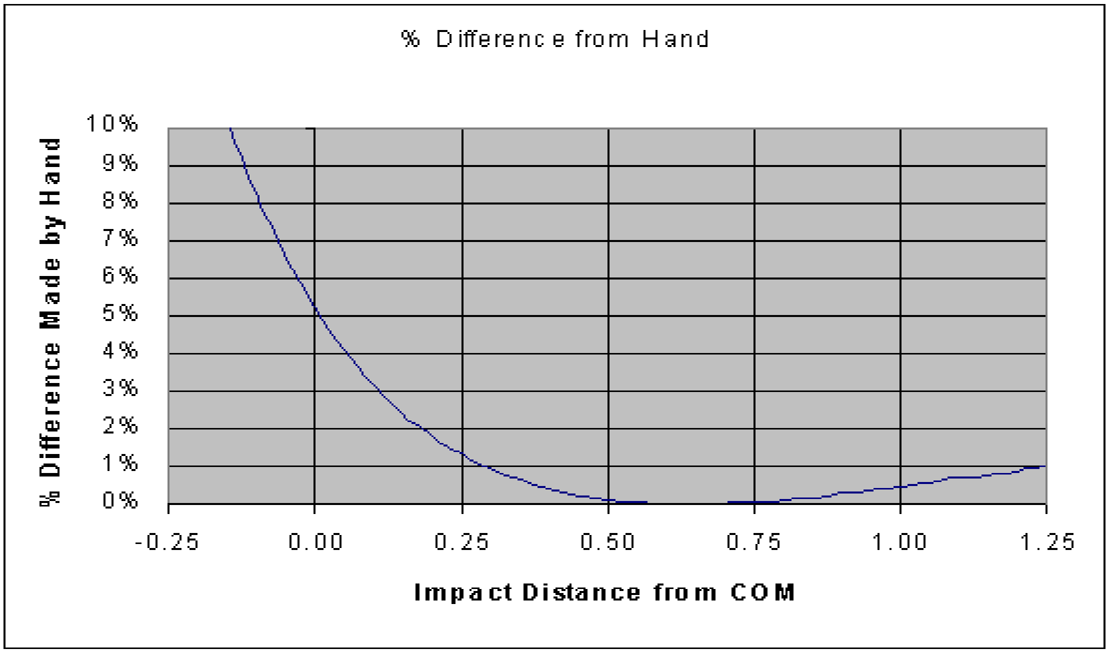
Intuition might tell you that any impact into the hand must dissipate energy, so all hand impacts will drain energy from the cut. In fact, you might think this is the primary source of energy loss in a cut. At one point that's what my intuition told me. But on further reflection I concluded that if the hand is kicked forward by an impact past the percussion point, then the hand's kinetic energy has been increased, which must mean less energy left in the blade, so less energy is available to damage the target. But if your hand is kicked backwards, its forward velocity, and thus its total energy, is decreased, so the hand must be passing energy into the blade, which would help drive the blade into the target. Running the numbers shows something different, but I bring up these intuitive concepts because in trying to understand historic swordsmen, we will also need to understand any likely misconceptions they may have had.

Here's a chart comparing a multiple impact hand model to a simple impact, which doesn't include any influence from the hand. It represents a single-handed cut, and as you can see, the hand increases the dissipated energy for all points save the hand's percussion point. However, this increase is quite small, especially anywhere remotely close to the hand's percussion point, which in this chart is located at 0.64 meters from the center of mass. The model was built with three



pairs of impacts, each of which consists of an impact with the target followed by a correcting impact with the hand. Each of these impacts was completely inelastic, and of the same delivered impulse to the target. These impacts were then followed by a finishing impact that completed the match of blade and target velocities, which was again followed by a final impact with the hand, which completed the motion.

The following graph is a closer look at the difference between the two curves.



As the graph shows, even at significant distances from the percussion point, out near the tip the hand contributes one percent or less to the energy dissipated into the target. In part, this is because the center of the lone hand was very close to the forefinger's percussion point, located at 0.73 meters from the center of mass. In general, there isn't much blade length past the forefinger's percussion point, and on swords with correctly sized pommels there really isn't any significant blade past that point. Only as you start moving the impact point closer to the hand, more than halfway back down the blade, does the hand even approach a one percent contribution to the impact. But as I've previously shown in the section on an impact's variable elasticity, back near the hand the impact is very elastic, and won't do much damage. The hand's contribution becomes large only as the impact point starts moving very close to the hand, in which case the result is similar to a punch delivered with a very, very heavy set of brass knuckles.

Building an Impact Model

Before we add another hand to the equation, let's take a closer look at the influence of the hand's impact with the sword. First, let me set up the physical parameters of the simulation. The hand is being modeled as a simple mass of 0.6 kg. I arrived at this figure by taking a 0.4 kg hand, which would be a typical value for someone weighing about 150 lbs, and then adding in the inertia of the forearm when pushed at the center of the hand, perpendicular to the forearm. This is exactly the same technique and formula used earlier to calculate the inertia of a blade, where the formula is still just $I = \frac{1}{1/m + p^2/I_{COM}}$. I've used this calculation on the assumption that your arm will be relatively aligned with the blade at the moment of impact, which is by no means assured. But in general, I think it gives a fair description of what goes on. Given the variations in body type, anything from 0.5 kg to 0.7 kg ought to be a good representation of someone's hand. For more accurate results, the Internet makes available a variety of body models and spreadsheets, which both researchers and undergraduate students in biodynamics make use of. Since the future holds more advanced sword studies, which will boil down to biodynamics, you might as well get used to the literature and techniques of studying human motions.

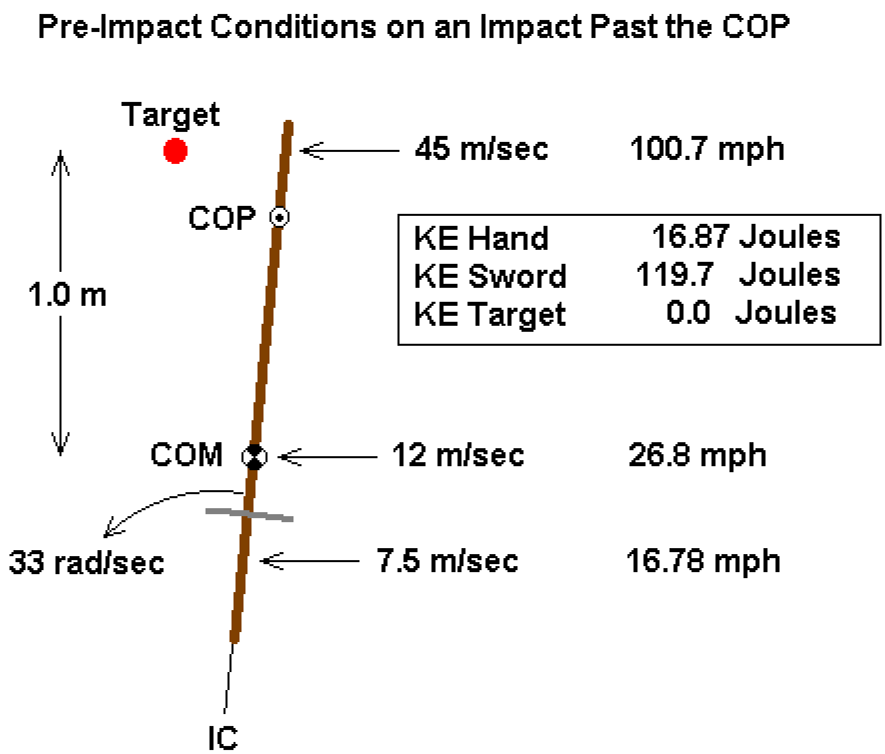
Here are the parameters used in the following analysis:

m_S	1.000	kg	2.20	lbs	mass of sword
x	0.120	m	4.72	inches	distance from forefinger to COM
handle	0.323	m	12.72	inches	distance from pommel to COM
Ix	0.102	kg-m^2	2.42	ft-lb^2	moment of inertia about forefinger
I_{COM}	0.088	kg-m^2	2.07	ft-lb^2	moment of inertia about COM
v_{COM}	12.000	m/sec	26.84	mph	velocity of sword's COM
ω	33.000	rad/sec	16.50	rev/sec	sword's angular velocity
m_T	0.500	kg	17.60	ounces	mass of target
xHand	-0.136	m	-5.37	inches	distance from COM to center of hand
mHand	0.600	kg	1.32	lbs	mass of hand
e	0.000				elasticity of target impact
vHand	7.500	m/sec	16.78	mph	velocity of hand

Note that the mass of the sword is a bit light. It is even almost impossibly light for our analysis of an impact occurring 1 meter past the center of mass. However, the model was built around a sword that has a percussion point at the tip, and to illustrate the effects of an impact past the percussion point, I've had to make a hypothetical impact far past the hand's percussion point. For realistic impacts occuring much closer in, the model should be reasonable. Also note that the angular velocity is very high. This actually occurs during high-speed swings, but this velocity can't be maintained for more than a short arc. It peaks near the target and then slows as your swing comes around. I don't think anyone can actually swing a sword around and around at 16 revolutions per second. In any event, the table sets up the parameters for the following impact analysis.

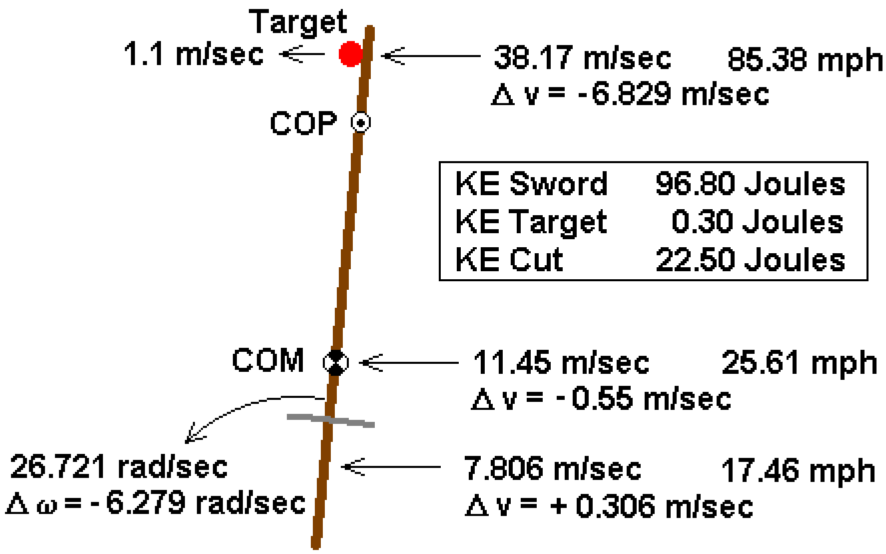
Impacts Past the Hand's Percussion Point

Below is an impact that occurs out past the hand's percussion point, labeled COP in the illustration. It includes the velocities of the relevant points on the sword, along with the sword's angular velocity. The initial energies of the sword, hand, and target are also provided.



As shown, the initial impact will have an edge velocity of 45 meters/second, which shouldn't be unreasonable for a real swing, based on what I've seen when running ARMA video clips through biodynamic video analysis software. Anyway, I'll let the collision be perfectly inelastic, and advance to the point where 0.55 Newton-seconds of impulse has been applied to both the target and the sword. After this initial contact, the target is travelling at 1.1 meters/second, and the edge has slowed to 38.171 meters/second. The sword's center of mass slows from 12.0 meters/second down to 11.45 meters/second, and the angular velocity slows from 33 radians/second down to 26.721 radians/second. This is illustrated in the next figure, where the change in velocities is also shown.

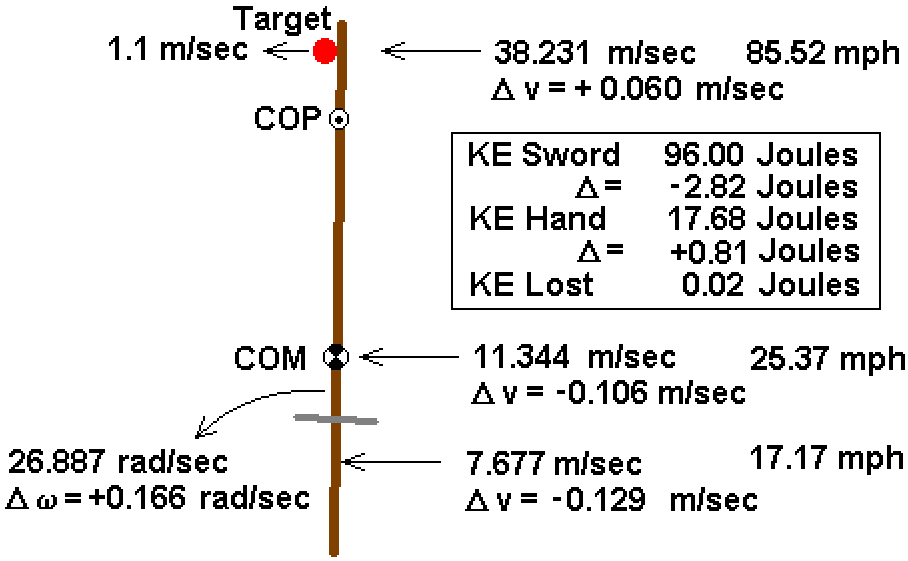
Velocities After Initial Collision with Target



If you look at the energy table, the blow is initially doing a very good job of putting energy into cutting the target, as opposed to accelerating it. You can also see that the edge is slowing down far, far more than the center of mass. This should be intuitively obvious, but people also tend to stop their hands shortly after impact, so some might think the blow stops the sword evenly. This is definitely not the case, and for an impact on your hand's percussion point, your hands aren't even slightly slowed down by the impact. For this particular impact, which is past the hand's percussion point, the velocity of the hilt actually *increases*! Recall the earlier section on finding a percussion point. Strike at a target, and if the handle kicks into your fingers, the impact is out past the hand's percussion point. If the hilt kicks back into your palm and wrist, the impact is inside the hand's percussion point. This is what occurs here. The hilt velocity, at the center of the hand, has increased by 0.306 meters/second.

Now we add the hand to the equation. Up to this point, we've let the hand continue travelling at 7.5 meters/second. But the hilt has now accelerated to 7.8 meters/second, so we have a collision where the handle impacts the fingers, driving the hand forwards. As a result of this collision, the hilt slows down, as does the swords center of mass. This also causes a loss of total sword kinetic energy, which you might think would weaken the blow. However, even though the sword loses energy, some of which is dissipated in the hand collision, while some is used to accelerate the hand, this doesn't mean that the target loses out.

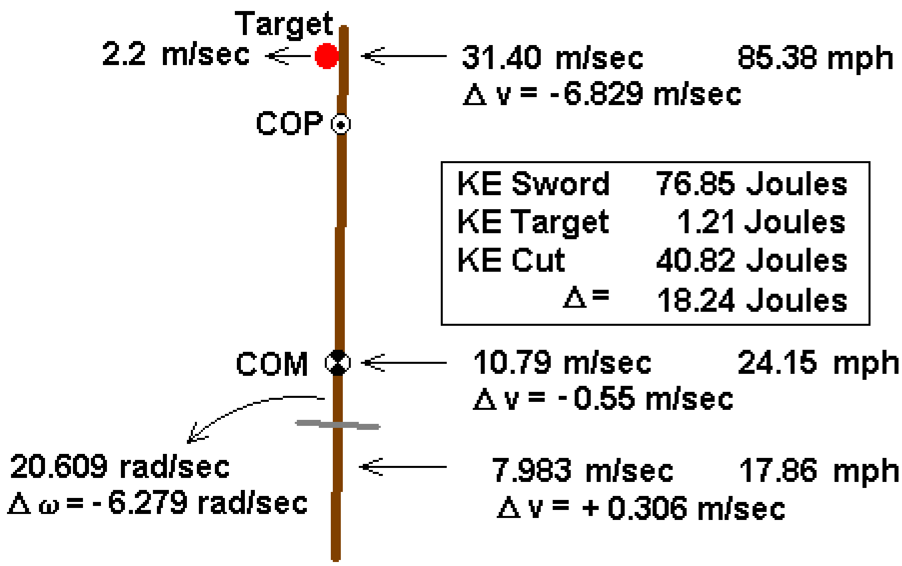
Velocities After Initial Collision with Hand



As the numbers show, the sword has lost 2.82 Joules of energ. The hand has gained 0.81 Joules, while 0.02 Joules was lost in the collision between the hilt and the hand. The hilt velocity has dropped, as has the velocity of the sword's center of mass. The collision with the hand has also spun the sword around a little faster, so the angular velocity has increased. This is intuitively correct, since an object that undergoes an off-center collision will have to rotate around the point of collision, to some degree or another. Imagine that the sword was traveling sideways, with no rotation. If the hilt collides with something, the inertia of the sword will cause it to rotate, trading away some of its linear velocity for angular. What is key about the collision is that it occurred at the hand, and as we've well studied, a collision at that point will cause a new motion, pivoted at the hand's percussion point, to be superimposed on the old motion. So all points below the hand's percussion point, labeled COP in the illustration, the velocity is decreased by the collision with the hand. For all points past the COP, the velocity is increased. In this collision the edge velocity at the impact point has been increased by 0.06 meters/second.

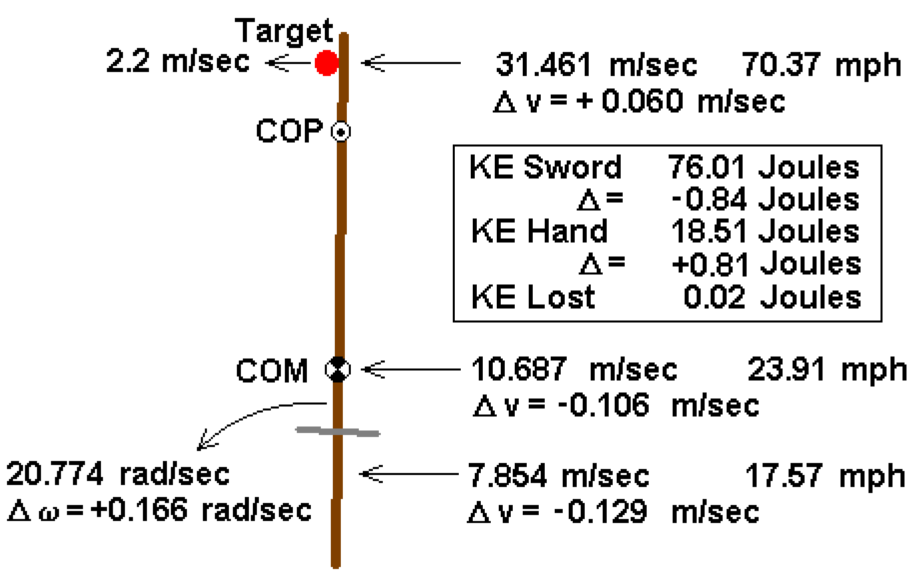
If we just stop here, we've already shown that collisions with the hand, for impacts past the hand's percussion point, serves to increase the edge velocity at the impact point. The result of the collision with the target is a function of the sword edge's inertia and velocity at the impact point. Since the edge velocity was increased due to the hand collision, the target collision will do more damage to the target, since a higher edge velocity does more damage than the slower edge velocity existing prior to the hand impact. And all this is because of that nasty hand shock we all sometimes feel. In essence, our hand acts like a poor pomel, but is forced to transfer energy at a rate higher than flesh should, which hurts. In the case of an impact past the percussion point, our fingers have absorbed energy from the blade, yet also caused the blade to dump more energy into the target. An electrician might say we've shorted out the blade, causing it to dump energy out both ends.

Velocities After Second Collision with Target



Here is the result of the second collision with the target. I've paused this impact after it delivered 0.55 Newton-seconds of momentum to the target, so the target velocity is exactly twice what it was after the first impact. This also means that exactly 055 Newton-seconds of momentum have been delivered to the sword edge. The key thing to note about this illustration is that the change in velocity at all the displayed points, and the change in angular velocity, is *exactly* the same as for the first impact. Since the geometry hasn't changed, and the sword's properties haven't changed, the same applied impulse produces the same outcome.

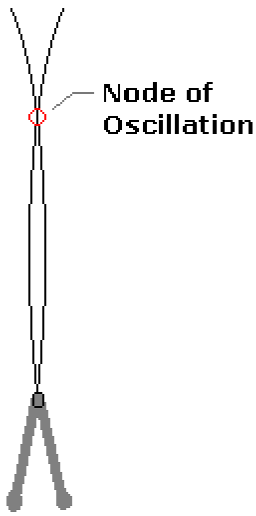
Velocities After Second Collision with Hand



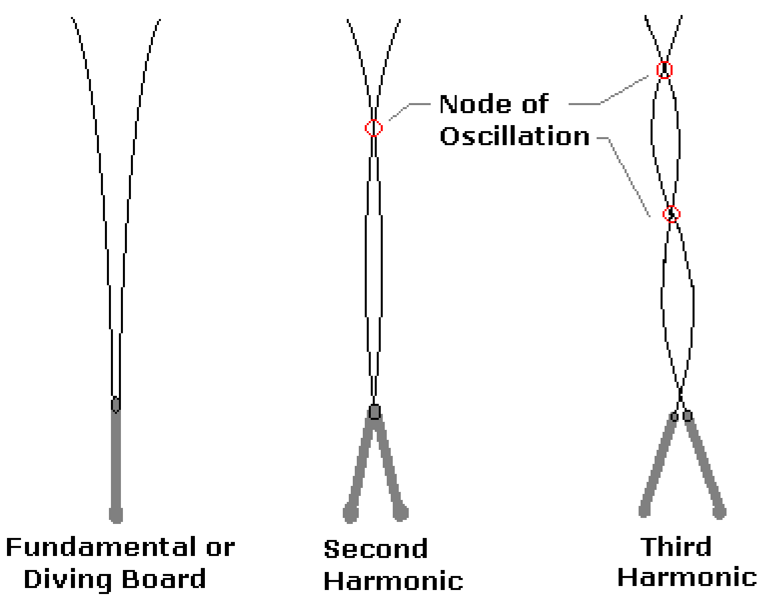
Similarly, the second impact with the hand produces identical changes to the first hand impact, as illustrated above. So this means that it doesn't matter how finely you divide up the impact analysis, how many times you let the impact model go from target to hand, the result is exactly the same. This bears some thought. Recall the earlier description of hand shock, and how the hand reaction force is a linear function of the impact force times the ratio of the distance that the percussion point was missed, relative to the total distance to the percussion point.

Chapter 11
The Modes and Nodes of Vibration

Let's look at the ways a sword vibrates. The diagram we most often see is this one.



The node of oscillation is presented as the best part of the blade to strike with because it presents the least shock to the hand, which robs the cut of energy. The problem is that this is not true. As we've seen in the sections on the percussion point, the location of least shock is going to coincide fairly closely to the true percussion point. What we're looking at here is the node of second harmonic vibration of the sword. There is also the fundamental frequency and the third harmonic to contend with, but you don't often see those drawn. For completeness here they are.



The reason you don't see these other two oscillations show up very often is that your hand quickly damps these modes. The third harmonic is also much harder to excite than the second harmonic, but even in baseball bat design all three figure in the analysis. So why do we focus in on the node of second harmonic oscillation? Because we can easily both excite it and see it. But the reason we can see it is that in a medieval sword, the hand normally doesn't damp it. If we want to minimize hand shock, we needn't worry too much about second harmonic oscillation. It doesn't make it into the hand anyway, which is why we can see it so clearly! Diving board oscillations are hard to see, since they're damped so quickly. If you want to see side-to-side diving board oscillations, place your hilt in a vice and tap the sword's tip. It will wobble back and forth, at a very low frequency, probably five to ten hertz, depending on your blade. Remember, that when you have a vibrating string, you can freely touch the nodes, without damping the vibrations. If you touch an anti-node, where the peaks of the waves are, you almost instantly damp the oscillations. If the blade was designed to minimize hand shock, and had been carefully crafted to locate the hilt's node of second harmonic oscillation so that energy is not passed to the hand, then we have no information about the maker's desired best impact location, other than it is probably not located at the other node of second harmonic oscillation. By locating the node at the hilt, we are doing the opposite of what has been supposed. We are reducing the hand shock from the second harmonic, even if that harmonic is heavily excited by impact. So, if we strike so that the second harmonic is not excited, we don't feel anything, as there isn't any vibration. But if we strike so that the second harmonic is heavily excited, we still feel nothing, as these vibrations aren't making it into the hand. So the blade maker was going to great trouble to eliminate the passage of second harmonic oscillations, which he probably wouldn't have done, if swords were used so as not to excite these oscillations anyway.

Energy in the standing waves

But even if this energy doesn't quickly go into hand shock, it gets left in the blade. Shouldn't this be avoided as a complete waste? Yes, but only a small amount of energy is in this second harmonic oscillation. You could set boundaries on how much by dropping a small weight on the blade, to excite the same amplitude that your normally see after a cut. By calculating the potential energy in the weight as $PE = mgh$, where m is mass in kg, g is gravity (9.8 m/sec^2) and h is the height (in meters), you can set an upper limit on the energy in the blade, measured in Joules. (1 Joule per second is a watt, the unit you use to buy light bulbs). There will be losses in transferring the energy into the blade, but for the most part you'll see that only a couple joules of energy are in the blade, out of hundreds of joules in the swing. Hitting far from the swords true percussion point and losing 20 or 30 percent of the swing energy is a bad way to save a percent or two of energy left in the blade.

Another visualization of the energy in the side-to-side oscillations comes from actual sparring practice. When you perform a deflection on a high-energy strike, with your flat, what happens to your sword? Not the strikes done with control, but the ones done with emotional intent. When you deflect a real blow, and don't get your angles right, so that your move resembles a static block with the flat, instead of a redirection, your sword absorbs a good deal of the energy present in the oncoming blow. If the sword is excessively flexible, it ends up flapping like a bird. When you look at the amplitude of these oscillations, which are primarily, if not almost completely, second harmonic side-to-side oscillations, you find that their peaks are easily 25 to 30 centimeters (about 10 to 12 inches) from the neutral centered position. So their peak-to-peak amplitude is 50 to 60 centimeters. The interesting information contained here is that the amplitude of these oscillations is proportional to the square of the energy they contain. A ten-centimeter amplitude of oscillation contains one hundred times the energy of a one-centimeter amplitude. Also note that these big impacts do not increase the frequency of the second harmonic oscillation. Greater impact energies increase the amplitude of the oscillation, not the frequency of a particular harmonic, and the higher harmonics are hard to excite.

Strike some blows against real objects, and again, try to quantify the peak-to-peak amplitude of the oscillations. Even with bad strikes, it will probably rarely exceed three, and almost never exceed ten centimeters. With good strikes it will rarely be more than two centimeters. Now, if you absorb a real strike from your partner on your flat, even with the transfer losses inherent in that process, and you get a 50 centimeter peak-to-peak amplitude, and your own bad strikes on targets give you even a five centimeter peak-to-peak oscillation amplitude, then the square law nature of the energy content says only 1% of your real strike's energy is being left in the blade. Some smiths, of late, have emphasized the importance of the location of the node of second harmonic oscillation. Unfortunately, they've been giving it more than its due. In large part I would blame the nearly complete absence of correct percussion point theory in the twentieth century sword community. Without understanding the percussion point, the sword's behaviors can be hard to explain. The second harmonic oscillations, side-to-side, are very low frequency. Generally they will be in the 5 to 10 hertz range, though some stiffer blades will be higher, maybe up to 15 hertz or more. The trend currently circulating is this, to try and explain hand shock with this second harmonic oscillation. It can't be done. Since you can yourself even maneuver a sword in the 5-hertz range, this frequency band would not constitute shock. To constitute shock, you should at least be in the 30-hertz to 100-hertz range, or higher. In baseball bats, the sting is above the hundred hertz range. If the second harmonic is in the 5-10 hertz range, the fundamental is in the 2.5 to 5 hertz range. One might think that a really big impact will cause you to feel the sting at higher frequencies. It is not true. Even a large impact cannot change the frequency of second harmonic oscillation. It will certainly increase its amplitude, but it won't significantly alter its frequency. Put simply, it's the same tone, only louder.

Energy in higher harmonics

If we talk about the third harmonic of oscillation, we're still only going to be in the 10 to 15 hertz range. To cause stinging, you need to get significant energy at least into the 60 hertz range. To do that, we'd have to excite at least the 12th harmonic of oscillation, possibly up to the 20th, or beyond. To excite these higher harmonics would also involve exciting all of their sub-harmonics, and additional harmonics as well. These oscillations are being excited by an impact, which is essentially an impulse function. The frequency spectrum will show spikes at the harmonics, but the energy in each spike will in general be much less than the spike of the next lowest frequency. Essentially, the lower frequencies will always contain much more energy than the higher frequencies. That's why big hits leave the blade flapping with intense second harmonic energy, but almost none in third or fourth. The blade doesn't end up looking like a guitar string. If you tried to put significant energy into the higher harmonics, the resulting energy in the lower harmonics would be sufficient to destroy the blade. As I've taken severe impacts, which don't show much discernable vibration in even the third harmonic, while the second harmonic oscillation contained enough amplitude to leave a permanent bend in the blade, I doubt explanations of blade behavior that require significant energy in extremely high harmonic oscillations are plausible. The maximum natural vibration energy contained in the higher harmonic's side-to-side oscillations doubtfully can exceed a tenth of a joule.

But you may wonder, what stings your hands when you deflect, if not these oscillations? The sword is a solid object. It's transferring the impact forces to your hand, unless the impact happens to strike your hand's percussion point. Your blade is simply passing along the impulse from the strike. The steel doesn't have to oscillate to do this. It merely has to hold together. It involves the speed of sound in the steel, not the blade's frequency when used as a tuning fork. As an aside, if you hit the steel with a projectile that exceeds the speed of sound in the steel, which in soft steel is about 5000 meters per second (11,200 mph), the steel atoms cannot provide mutual support to each other, and projectile penetration is much, much easier. You get to punch a hole through the metal plate, without the entire plate's deformations robbing energy from the projectile. But even with slower impacts, the plate's natural resonant frequency doesn't really come into play.

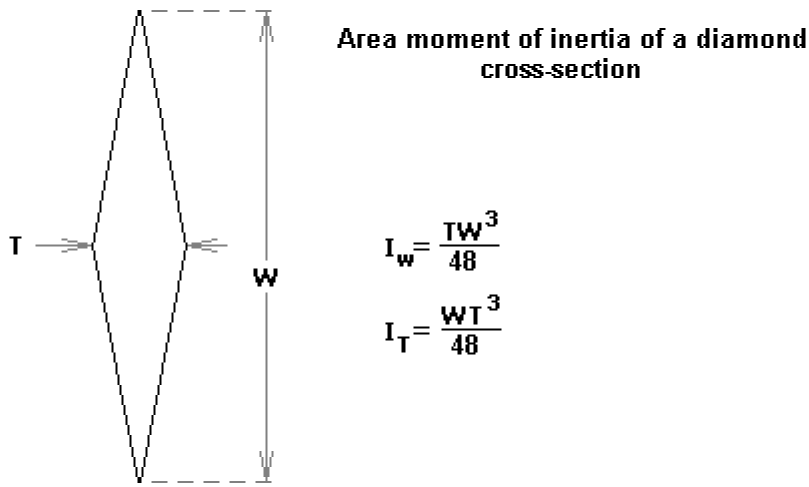
As an illustration of the fact that you're unlikely to excite these harmonics, without destroying the blade, let me relate a recent impact to my blade. In an experiment to illuminate a different sword property, my sparring partner delivered a blow with a ¾ inch diameter piece of rebar, about 4 feet long, with all his might, while I performed a static, horizontal block with my Del Tin 2151 (I said it was an experiment, not a combat technique). The Del Tin flexed past 100 degrees, with the pommel up, the hilt pointing down, and the tip of the blade pointing back up at the sky. I remember it because it's burned into my retina, since this amount flex allowed my opponent's weapon to continue through my defense, and impact on my leg. Yes, it was violent, and no, there was no sting, aside from my leg getting hit. The oscillation frequency was still down in the 3-5 hertz range, even though the blade was flapping like a bird. There was no oscillation to sting the hand, or cause loss of grip. If the impact had been slightly greater, it probably would have destroyed the blade, but my hands, and grip, would've been unaffected. So please note. No amount of side-to-side oscillations should knock the sword out of your hand. No amount of side-to-side oscillations will sting your hand, nor will they cause you to drop your blade. There are well-developed medieval techniques for disarming your opponent. Normal blade impacts aren't one of them. But if an impact on the flat is strong enough to flex the blade past ninety degrees, but still doesn't product any appreciable sting, then it's doubtful that sting can be produced in the side-to-side plane. Note, also, that I'm not referring to hand shock, which will come from the impact, according to the percussion point relations derived earlier.

Let me make a further note on the possibility of these higher harmonics existing in the blade, and thus the importance of controlling the node of second harmonic oscillation. If you've actually got the 12^{th} harmonic excited, then in all probability you have 2 nodes from the second harmonic, three nodes from the 3^{rd}, four from the 4^{th} and 5^{th}, each, six from the 6^{th} and 7^{th}, etc. You end up with about 85 nodes strung out along the blade. Now just which is the important one, and don't you have at least a half dozen nodes just under your hand? I say that the second harmonic is still the primary one, as the others are hard to excite. However, the second harmonic won't explain sting and shock, without all the other harmonics being of great importance. But these high harmonics are not important, as they only show up far past any energy that would destroy the blade. Furthermore, if you've every seen how music looks on an oscilloscope, you can form a mental picture of what the blade shape would be, if these higher harmonics were visible. Picture what would result from getting the blade red hot, and then scrunching it up into a folded little waveform, like curtains hanging from a window, only much more irregular. Thus, the theory that control of the side-to-side nodes of second harmonic oscillation controls hand shock is, I think, incorrect.

Velocity of the side-to-side waves

There's just one more important point to bring out, which is the travel velocity of these side-to-side waves. Measure the distance between the two nodes of second harmonic oscillation. This measurement is one half the wavelength of the second harmonic, since it's a half-wave, not a whole one. A whole one won't fit on the blade, but is still twice the length of the half-wave, and is given the letter λ, for length. Now count how many oscillations per second the blade vibrates at, which is f, for frequency. The transmission speed of the wave is given by the formula $c = \lambda \cdot f$, which should be instantly familiar to anyone involved in physics, radio, optics, electronics, music, and, well, just about everybody who's had a science class. Interestingly, on most swords the wavelength of second harmonic oscillation probably varies from about 0.5 to about 0.75 meters, being 2 times 2/3 of the blade length, or 1.33 times longer than the blade. The frequency varies much more, being dependent on blade stiffness.

There is a crude formula to estimate the speed of a mechanical shear wave, which we can use to form a rough guess on side-to-side frequency, plus the elusive edge-to-edge frequency. It's given as a function of the area moment of inertia of the cross section, the density of the material, and the length of the section. If we assume a uniform, diamond cross-section, then we can use a standard formula for the area moment of inertia. Note that this is not the mass moment of inertia we have been dealing with so heavily. It is an area moment of inertia, which is used to calculate the stiffness of beams, among other uses.



For the side-to-side oscillations, we look at the side-to-side area moment of inertia, given as $I_T = \frac{W \cdot T^3}{48}$. For the edge-to-edge oscillations, we'll use $I_W = \frac{T \cdot W^3}{48}$. Let's use a piece of steel with a density σ of 7.85 g/cm^3 and a modulus of elasticity of Y=213.9 Gpa, or 2.14e12 dynes/cm2. The blade thickness is 3/16", and its width is 1 ¾". Interestingly, for this run of the mill cross-section, the edge-to-edge stiffness is 100 times the side-to-side stiffness. Now, for a section of length L, one formula for the frequency is $f = \frac{9 \cdot \pi}{8 \cdot L^2} \cdot \sqrt{\frac{Y \cdot I_W}{A \cdot \sigma}}$, where A is the area of the cross section, and in our diamond case $A = \frac{T \cdot W}{2}$. The formula isn't going to give a perfect answer, as shear waves have a few added complexities, but it will be close.

If we convert everything to a common set of units, we get the following results. For a 48" long blade, the side-to-side frequency is 11.6 Hz, while the edge-to-edge frequency is 116 Hz. The wavelength in either case is $\lambda = \frac{4}{3} \cdot L$, or in this case 1.6 meters. The calculated velocity of the side-to-side wave is 18.6 meters per second, and for the edge-to-edge wave the velocity is 186 meters per second. For an impact near the tip, the side-to-side wave would take about a sixteenth of a second to reach your hand. At the angular velocities that swords typically reach, the blade would travel through 60 to 90 degrees before the side-to-side wave hit. The edge-to-edge wave would arrive 10 times faster.

When you strike a target, or get struck during a deflection, these side-to-side oscillations are taking a long time to reach your hand. A very long time, from maybe 0.04 seconds to 0.12

seconds, depending on the blade and impact point. There is no way you'd really even associate the feel of these wobbles with the actual impact. Given the rotational speeds that we move these swords at, you'll be through the deflection and either recovering or into the counter attack before the oscillation hits your hands. What you'll probably feel is a sharp impact, followed noticeably later by a low frequency wobble in your hand. Another problem with the side-to-side oscillations is that the wave doesn't change the final direction of the blade in a significant degree. We've all had our swords knocked around, but a mere sine wave won't do this. While the hilt is riding the wave, it's direction does change, but changes around the initial direction it was pointing, always returning to the original orientation. Unless the oscillation is damped during one of the extreme excursions, the wave can't explain your blade getting knocked aside.

The Edge-to-Edge Oscillations

However, there's still one problem with all this analysis. It's being done in the wrong plane! The side-to-side oscillations are low energy, and low frequency, compared to the oscillations occurring from edge-to-edge. But nobody seems to bother with the oscillatory node locations occurring edge-to-edge. This is probably because even though you can feel them as a sting, after a hard strike, it's extremely hard to see them. We can sometimes hear them as a nasty rattling sound, or buzz, down in the hundred hertz range, if part of our hilt is loose. Generally the method of mapping them involves some transducers and lab equipment. Unless we think of some trick (like pouring a bit of sand down the fuller, and tapping the edge to make patterns in the sand), we can assume that these nodal locations would've been unknown in practice.

We know that tip impact will excite the fundamental mode, but there's really nothing much to be done about it. In baseball bats, it's just a given that hitting too close to the tip will excite the diving board mode of oscillation. The only solution is to make the blade very stiff edge to edge, and the blades that remain broad, all the way to the tip, seem to follow this design approach. The second harmonics are considered less important than the fundamental, which is why the sweet spot of a baseball bat is not moved to the tip (It's been tried experimentally, and isn't as comfortable), and these second harmonics seem to be handled, somewhat, by the hilt location. The third harmonics are generally the least important, and since we currently don't have a good grasp of the fundamental, I wouldn't worry about the others at this time. Baseball bats don't show much effect of these oscillations, until the ball hits far from the percussion point. The physicists show graphs of the stiff bat model versus the actual flexible bats used. Some of this data may be of value to blade smiths in understanding the effects of edge-to-edge blade flex on striking. However, the stiffer the blade is, in this edge-to-edge direction, the less effect will any of the modes of oscillation be in a real impact. With an extremely stiff blade, the oscillation modes can be completely ignored, whereas the impact location relative to the percussion point, being based solely on mass distribution, not mass and stiffness distribution, still retains its supreme importance.

Since side-to-side oscillations can't constitute the feeling of hand shock, or sting, another effect must be the cause. Take it as given that the oscillation frequency, and node location, of the sword is determined by the mass versus stiffness profile, and that the mass profile is the same in either the edge-to-edge or side-to-side planes. In short, mass is mass, in either plane. As the stiffness profile is much, much larger in the edge-to-edge direction than it is in the side-to-side direction, the oscillation frequencies in the edge-to-edge direction are very, very much higher than the side-to-side ones. So the side-to-side oscillations can't be pushed up into the hand shock frequency band, at least not without blade destruction, and the edge-to-edge oscillations have low order harmonics already in this frequency band. Additionally, in any strike, the impact angle of the blade will dump vastly more energy into the edge-to-edge plane, than it will into the side-to-side plane. Thus, it stands to reason that any oscillation component of hand shock is best explained by the more probable excitation of the edge-to-edge plane's fundamental, second, and third harmonics. The excitation of the side-to-side plane's 12th or higher harmonics is extremely improbable.

Furthermore, even if the side-to-side plane's 12th harmonics or higher were excited, and significant, the resulting hand shock would not be controllable by simply adjusting the second harmonics node location. Please note the earlier pictures of the oscillation modes. The second harmonic has two nodes. The third harmonic has three nodes. The 12th harmonic will have 12 nodes. It's like a guitar or piano string. For the higher harmonics of the string's fundamental frequency, you will have a large amount of nodes. By the time you've got 12 nodes spaced on a 48", two-handed sword, you've got a node about every 4 inches. You are virtually guaranteed to have a node in the hand. But it won't do you much good. If part of your hand is on the node, the rest is certainly not. You'll have one finger shock free, and the rest get stung. Thus, it is extremely unlikely that controlling the node location of second harmonic oscillation in the side-to-side plane will in any way decrease perceived hand shock resulting from impacts.

Continuing, another part of the harmonic balance theory is that the second harmonic's node location, out on the blade, is under the complete control of the blade smith. It is not. The smith has only limited ability to move the node's location. In Del Tins product line, of 23 blades (disregarding 2 outliers, for which the data is suspect, given the standard deviation of the measurements) the node location varied only 9.5% as a function of blade length. If controlling the node location is so crucially important, why is one of the world's largest medieval sword manufacturer's entire, and very diverse, product-line relatively invariant in this regard? The truth is, the second harmonic's node location can't be moved very much. Unlike the percussion point, whose distance from the center of mass can vary anywhere from $\sqrt{\frac{I_{COM}}{m_{SWORD}}}$, all the way to infinity, the second harmonic's node location is constrained to a very narrow blade area. Anything the smith does to move it any great distance will have a significant effect on either the tip's thickness, strength, or overall moment of inertia. Since the tip (by which I refer to the last third of the blade) is really the important part of the sword, screwing up its performance, for a slight change in oscillation behavior, in the side-to-side plane, is an exercise in engineering that is of a very dubious nature.

Bear in mind, that since the stiffness profile in the edge-to-edge plane, versus the side-to-side plane, is very different, the node locations in these two planes is going to be somewhat different, as well. However, if the sword maker is keeping the basic cross section of the blade constant, and varying its size, then the edge-to-edge and side-to-side stiffness profiles will at least be somewhat correlated. By trying to control the side-to-side node location, moving it to the hilt, the maker might also be moving the edge-to-edge node closer to the hilt. This may improve the feeling of vibrations in the edge-to-edge plane. As argued earlier, if blade oscillation has any significant bearing on hand shock, then it is probably the edge-to-edge oscillations that matter. It is the edge-to-edge oscillations that should have their node locations carefully placed, assuming that these are the oscillations that contribute to hand shock, and rob the impact of energy.

Another likely possibility is that by trying to improve the sword's feel, and performance, they are accidentally getting the percussion point closer to its proper location. If they are adding more distal taper, and slightly beefing up the strong section of the blade, then the sword's percussion point is moving back toward the hilt. Since most people have been taught to strike using the second-harmonic's node location, they are striking inside the percussion point of a medieval sword. As most reproductions don't incorporate much distal taper, giving them a moment of inertia that's far too large, and also don't have thick blades near the hilt, their percussion point location will be out too far. Combining this with a large pommel to improve the sword's “balance” will, in fact, probably move the percussion point way off the end of the tip. (I own one of these). Anything that brings a reproduction sword’s percussion point back will undoubtedly improve its feel, at least for strikes on the second harmonic node location. I think the nodal theories may be helping to improve reproduction swords, but only by accident.

In closing, the whole oscillation theory is open to question. The oscillations that are being debated are certainly of less importance than other oscillations, which are not even being studied. Possibly, some blade smiths will investigate these oscillations, and present some good data on their magnitudes and effects. I can imagine a quick study, where a sword is swung against an aluminum foil covered target in a darkened room. Touching the foil closes an electronic circuit, which will fire an electronic flash unit, illuminating the flex for capture by a camera. The camera will have it’s shutter locked open for the duration of the swing. An adjustable timer placed in the circuit to delay the flash would allow a series of photos to record the entire flex behavior in good detail. It would make a very good web posting to show how a blade bends, in the edge-to-edge plane, during hard impact. If someone wants to spend some money then a high-speed camera, probably capable of 10000 frames per second or more, would be a very good analytical tool to use on this problem. Additionally, if we place some transducers on the blade, we can directly record the vibration data.

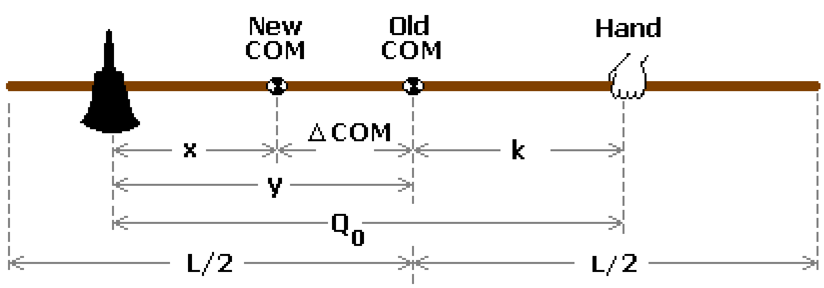
Chapter 12
Pole-arm Design Theory

Given what's been learned so far, how do you apply this to design? How do you go from a set of user specifications, and come up with a weapon that fulfills them? How do you know how much distal taper? How do you size a pommel? How do you size a cross guard? How do you know where to add the head to a pole-arm? In short, how could a medieval weaponeer come up with new designs? Let's start with the basics, and then move forwards.

Impact Pole-arms

Since we've already covered the staff, let's figure out where to put a pointy thing on it, such as an axe head.

One method is to just add the axe head, and then figure out where its corresponding percussion point ended up. In this case I'm talking about the axis of rotation, relative to the axe head, where you would want to place your hand. This method has some merit, since it's so easy to do. However, what if you already know where you want to place your hands? Then, you need to know where to place the axe head, in order to ensure that the hand location will be correct. This looks like it will involve a lot of math, but some surprises await us. To begin with, let's look at some of the measurements involved.



First, we need to find ΔCOM, which is given by the formula $\Delta COM = y \cdot \left(\frac{m_{HEAD}}{m_{HEAD} + m_{STAFF}} \right)$, where y is the distance from the head to the shaft's old center of mass location, and the masses are as indicated. Then we have to find the percussion point, relative to the head location. To do this, we need to know the new moment of inertia. The old moment of inertia, computed about the middle of the staff, is simply given by the formula $I_{COM\_OLD} = \frac{m_{STAFF} L^2}{12}$. To simplify things, let's treat the head as a point mass, having no moment of inertia of its own, which is not really a valid assumption. If you try to compute the moment of inertia about the shaft's center of mass, and then track the changes, the math gets very ugly. The problem is still simple to solve with a calculator, summing the terms as you go, but the equations get large if you go for a single algebraic solution. The problem is far easier, if you compute the percussion point about the new head.

The old percussion point location, measured from the future head's position is just $Q_Y = \frac{I_{COM\_OLD} + m_{STAFF} y^2}{m_{STAFF} y}$. This is from the head's perspective, as if it was actually the grip.

To find the moment of inertia about the head's future position, we just took the moment of inertia about the old center of mass location, which was the center of the staff, and used the parallel axis theorem to find the plain staff's moment of inertia, when at distance **y** from the center. The percussion point distance includes the distance from the hand to the shaft's center of mass, so it's measuring the distance from the hand to the hand's percussion point.

Now we just add the head, and see what happens. The head, of course, has its own moment of inertia, about its own center of mass, so we'll call this I_{HEAD}, but since its center of mass is being added at point y, we can just add this inertia to the shaft's inertia about point y. Then we adjust the mass to reflect the addition of the head, and adjust the distance from the head to the new center of mass.

$$Q_{HEAD} = \frac{I_{COM\_OLD} + m_{STAFF} y^2 + I_{HEAD}}{(m_{STAFF} + m_{HEAD})\left(y - y\left(\frac{m_{HEAD}}{m_{STAFF} + m_{HEAD}}\right)\right)}$$

Here, the term $\left(y - y\left(\frac{m_{HEAD}}{m_{STAFF} + m_{HEAD}}\right)\right)$ is really just $y - \Delta COM$, where

$\Delta COM = y\left(\frac{m_{HEAD}}{m_{HEAD} + m_{STAFF}}\right)$, as before.

But the bottom term in the equation simplifies quite a bit, giving us

$$(m_{STAFF} + m_{HEAD}) \cdot \left(y - y \cdot \left(\frac{m_{HEAD}}{m_{STAFF} + m_{HEAD}}\right)\right) = y \cdot \left[(m_{STAFF} + m_{HEAD}) \cdot \left(1 - \left(\frac{m_{HEAD}}{m_{STAFF} + m_{HEAD}}\right)\right)\right]$$

This simplifies further, to just $y \cdot [(m_{STAFF} + m_{HEAD}) - m_{HEAD}] = y \cdot m_{STAFF}$.

So the whole equation for the percussion point becomes

$Q_{HEAD} = \frac{I_{COM\_OLD} + m_{STAFF} y^2 + I_{HEAD}}{y \cdot m_{STAFF}}$. Amazingly, except for the addition of the term I_{HEAD}, this is exactly the equation for the percussion point location before the head was added! <u>If the head were a point mass, it wouldn't affect the percussion point location at all, just as we found for the pommel's affect on the percussion point, relative to the pommel, itself.</u> So this can be simplified further, to the original percussion point distance, plus a small error term. This boils down to the following.

$$Q_{HEAD} = Q_Y + \frac{I_{HEAD}}{y \cdot m_{STAFF}}$$

Here, we have the new percussion point distance as the old percussion point distance, plus an error term, which is just the head's inertia about its center of mass, divided by the distance from center, as shown in the drawing, and the mass of the staff. Since I_{HEAD} will typically be very much smaller than the shaft's inertia, about the head location, it can be ignored for most cases. All that will happen is that your hand location will move back, probably by only a few inches. But for accuracy, I must bring up an important point. The error term isn't the extra distance you must move the head, displacing it from the initial percussion point. It's the extra distance that your hand moves toward the end of your shaft. Since the mapping from hand positions to corresponding percussion point locations is non-linear, you can't just add this error term in when figuring out where to place the head.

To figure out the true placement location for the head, so as to leave the hand's location unchanged, we have to approach the math a bit differently. The moment of inertia of the pole arm, after the head is added, can be computed by adding the moment of inertia of the head to the moment of inertia of the shaft, computed about the head's center of mass location. This is a snap, using the parallel axis theorem, and ends up with the following.

$$I_Y = I_{SHAFT\_COM} + m_{SHAFT} \cdot y^2 + I_{HEAD\_COM}$$

Next, we use the percussion point equation, in the form that gives the distance to the percussion point from the axis of rotation, not from the center of mass. This is done by using the moment of inertia computed about the axis of rotation, instead of the moment of inertia about the center of mass. So we have

$$Q_0 = \frac{I_Y}{(m_{SHAFT} + m_{HEAD}) \cdot x}$$

But, looking back at the drawing, x is just $y - \Delta COM$, and we already know how much the center of mass moves. Substituting in, we get

$$Q_0 = \frac{I_Y}{(m_{SHAFT} + m_{HEAD}) \cdot \left(y - y\left(\frac{m_{HEAD}}{m_{SHAFT} + m_{HEAD}} \right) \right)}$$

But the y factors, and we get

$$Q_0 = \frac{I_Y}{m_{SHAFT} \cdot y}$$

Expanding this gives

$$Q_0 = \frac{I_{SHAFT\_COM} + m_{SHAFT} \cdot y^2 + I_{HEAD\_COM}}{m_{SHAFT} \cdot y}$$

Multiplying both sides by y gives us

$$Q_0 \cdot y = \frac{I_{SHAFT\_COM}}{m_{SHAFT}} + y^2 + \frac{I_{HEAD\_COM}}{m_{SHAFT}}$$

$$Q_0 \cdot y - y^2 = \frac{I_{SHAFT\_COM}}{m_{SHAFT}} + \frac{I_{HEAD\_COM}}{m_{SHAFT}}$$

$$y \cdot (Q_0 - y) = \frac{I_{SHAFT\_COM}}{m_{SHAFT}} + \frac{I_{HEAD\_COM}}{m_{SHAFT}}$$

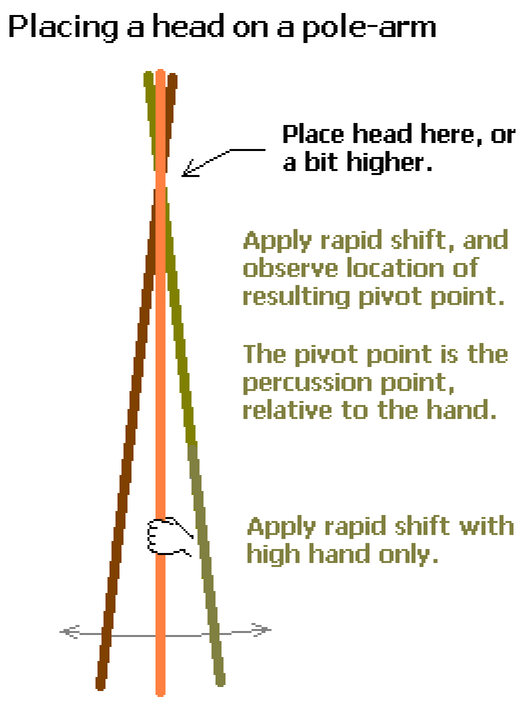
But, looking back at the drawing of the hand on the shaft, the value $Q_0 - y$ is simply k, the distance from the hand, which acts as the axis of rotation, to the old center of mass. Since this is already given as a design parameter, existing prior to adding the head, the equation for where to place the new head becomes very simple, indeed.

This is the pole arm equation. $$y = \frac{I_{SHAFT\_COM} + I_{HEAD\_COM}}{k \cdot m_{SHAFT}}$$

Note that this equation reverts to the old percussion point location when the inertia of the head goes to zero. This also breaks apart into the old percussion point location, and an error term that's slightly different from the previous error term, which told us how far back our hand would move, if we mounted the head at the old percussion point location. Whereas the hand would have to move back according to the formula $FudgeDist_{HAND} = \frac{I_{HEAD}}{y \cdot m_{SHAFT}}$, the head can be moved toward the other end, by the distance $FudgeDist_{HEAD} = \frac{I_{HEAD}}{k \cdot m_{SHAFT}}$. So let's be clear about the necessary adjustments. The equation for adjusting a given end's percussion point always includes the moment of inertia of the added object, calculated solely about that object's own center of mass. Then this is divided by the mass of the existing components, before the new object is added. Then the next division uses the distance, measured in between the existing component's old center of mass location, and the other end's percussion point. So, if you want to know how much to adjust the head, divide the head inertia by the shaft mass, and divide this by the distance from your hand to the old center of mass. If you want to know how much to move your hand, when the head is placed at the hand's old corresponding percussion point, divide by the distance from the head to the old center of mass.

Quick and dirty head placement

Okay, so you don't want to do the math, since it's not an authentic method of pole-arm design. How do you put together a pole-arm, in the middle of nowhere, with no math? The math showed the way, and now we can toss it aside. Note that the head was placed just a bit past the bare shaft's percussion point, relative to the hand. We know how to find the hand's percussion point, in only a few seconds. Recall the pivot test.



So, if you know where you want to place your hand, on the completed weapon, head placement becomes trivial. Just grab your shaft in the normal fashion, wobble it about, side-to-side, and observe the point on the shaft that's acting like a pivot. If your head is very small, even if it's very heavy, it won't have much moment of inertia about its own center of mass, and you can place it right at the observed pivot point. However, if your head is long, or very wide, like a long transverse spike, perpendicular to the shaft, then you need to move it a few inches, toward the business end, and away from your hand. This extra bit of distance will compensate for the head's own moment of inertia, computed about its own center of mass. You can see how simple all this is, once you know the tricks.

Another method of adjusting head placement would be to move your hand an inch or so, forward of its normal location. Then perform the pivot test, which will give a pivot point just a bit further out than it normally would. Place the head directly on this pivot location. Experience with different head types, and comparisons to a large variety and number of pole-arms, would have quickly given their makers a gut feel for these fudge factors. Also, by replicating a well-known and good placement location, on a particular type of shaft weapon, the makers would ensure a close tolerance.

But how do you know if you've got it right? Simple, just do the pivot test again, after you've placed the head on the weapon. Place your hand in the usual spot, wherever that may be for your style, and make sure the weapon pivots around the center of the head. This brings up a handy double check, and a refinement on the fudge factors applied earlier. Perform the pivot test, and then slide the head onto the shaft. Make sure the head squeezes the shaft just enough, so that it won't slide around, and perform the pivot test again. If the pivot location is out past the center of the head, slide the head further out. If the pivot location is closer in, below the center of the head, then slide the head a bit further in. In each case, just keep moving the head to the observed pivot point. You'll probably not have to do this more than twice, before you have the head positioned within less than a centimeter of perfection. Then just nail it down. Your design is complete. Simple, isn't it?

Another handy thing about this method is that it automatically covers any head configuration. In some cases, the desired impact point of the head is nowhere near the head's own center of mass, which is quite likely, considering the bewildering variety of head shapes. With the test and adjust method, there isn't a problem getting the placement right. You may have to adjust the head's location more, but you'll still get the exact spot, and in only a minute or two.

Flipping the pole-arm around

There are several ways to flip all this knowledge around, and these are quite interesting. The first, and most obvious, bears on existing, authentic, pole-arms. If the hand location determines the correct head placement, then an existing head placement determines the design's correct hand location. So if you pick up an authentic pole-arm, and find the head's corresponding percussion point, you have the proper hand location for delivering the cleanest blows. This could be referred to as the weapon's natural hand location. You might think of it as the "home" position, though in combat you may seldom, if ever, have one of your hands gripping the weapon there. Tactical considerations require you to hold the weapon in the best location for defeating your opponent, and these grip locations may move back and forth as opportunity and circumstance dictate. If you create an opening that gives you time to move to the "home" position, before delivering the blow, then by all means do so. But a pole-arm strike, delivered from anywhere near the hand's home position, will still have devastating effect.

As an aside, if the pole arm has a long, thin head, like a pole-axe, it has a larger moment of inertia around the head when rotated in the plane of the strike, than if rotated on an axis that passes from one edge of the head to the other. This can slightly throw off the percussion point determination, which won't be exactly the same in one axis as in the other. Meaning that waggling the weapon side-to-side, as in the simple pivot test for a sword, will give a slightly shorter percussion point distance than waggling front to back, which is the way an impact will truly move the weapon. To find the exact percussion point in the front-to-back plane, you might put a small rod through the shaft, perpendicular to the head, like the croix of a poleaxe. If you leave this rod sticking out about an inch or so from each side, you can use it just like the cross on a sword, and do a pivot test based on the croix. This will be more accurate than a side-to-side pivot test, holding the center of the edge of the axe.

Which hand determines the percussion point?

I know that swords base their percussion point on the knuckle of the top hand's index finger. Normal axes, in use today, base it off the bottom hand's pinky location. If you observe someone chopping wood, the top hand slides up, to apply more torque at the start of a strike, but then slides back down for the impact. The top hand also has a very loose grip during impact, which avoids absorbing the impact force. Pole-arms might be used similarly, in which case the percussion point, relative to the head, gives the bottom hand location. In either case, the percussion point will correspond to the hand location at impact, and says little about the hand location existing prior to impact, nor to the normal hand location during combat, in one of the guard positions. However, an axe has a very short handle, with the percussion point all the way toward the end, so only the bottom hand could possibly be near the percussion point. Pole arms use much longer shafts, and should have a good length of shaft existing behind the percussion point. If the bottom hand was at the percussion point, that would leave quite a bit of unused shaft. So I'll speculate that the top hand stays at the percussion point, but more research and experimentation are called for.

In a study of the Persians fighting Alexander the Great, there is mention of gaudy "Apple Bearers" who carried spears with spheres on their butts[1]. Since pommel literally means apple, in Latin, these might be spears with true pommels, which is an interesting idea. Then you could hold the spear nearer the back, yet still have a percussion point at the tip. Unfortunately, this idea apparently didn't survive contact with Alexander's forces, and I've never heard of another case of a spear with a pommel, though there might be some found somewhere. But for the concept to work, the spearmen were probably basing the percussion point on their top hand, since the other hand was probably on, or near, the pommel, and a pommel doesn't affect the percussion point relative to the pommel, itself. There's also the clear possibility that these pommels where merely wooden balls on the end, to provide a stronger grip for thrusting, much like putting the hand on the back of the pommel for many European thrusts. If you did have a style where you hold the spear by the butt, so you can pass it to your left and right with ease, you might want to add a ball so the butt can't get shoved through you while you're shifting the butt past your innards.

Iron Inlay Shaft Protection

If you're going to apply iron strips to the shaft, in order to protect it from being cut through, then this needs to be done prior to head placement. If the strips are uniform, and run the full length of the shaft, then they don't affect the bare shaft's percussion point locations. However, note the fudge factor we calculated earlier, $FudgeDist_{HEAD} = \frac{I_{HEAD}}{k \cdot m_{SHAFT}}$. This factor includes the original mass of the shaft, and the iron strips, and the slots cut for them, affect the shaft mass to a significant degree. So, although the shaft's percussion point behavior is unaffected by the iron strips, the extra distance you have to move the head will be decreased by their addition. You can get close to the proper head location without them, but some adjustment may be necessary

[1] "Carnage and Culture" – Victor Hanson, pg. 70

if you apply them later. So don't commit to a head location till the shaft's iron strips are taken care of. If the strips don't run the full length of the shaft, then they have a profound effect on the percussion point location, and even an initial check of the shaft's percussion point locations will be useless. So apply the wrought iron first.

Chapter 13
The Cross-Guard?

The cross guard is an interesting piece of the sword. John Clements has sometimes expressed consternation at divining its purpose, which is actually what got me started on this whole analysis of sword properties. In looking at the manuals, one might conclude that its main use is to provide a convenient gripping point, so your opponent can rip your sword from your grasp. It can certainly be used for a few interesting moves, but are these its purpose or just very convenient uses? Would these uses have been enough to let it remain on medieval swords for century after century? We all, of course, started with the obvious notion, passed down to use by the legendary researchers of fencing, that the guard is there to protect your hands. We are also told that it is but a very crude and primitive form of hand protection, suited only to the crude untutored hacking methods of the medieval knights. But we find few dinged up guards, *and the manuals don't show anyone blocking powerful blows with the cross-guard.* It is used in the bind, but it doesn't need to be very large for this. The Greek and Roman cross-guards are very much smaller, as are many swords found throughout Europe prior to the middle ages.

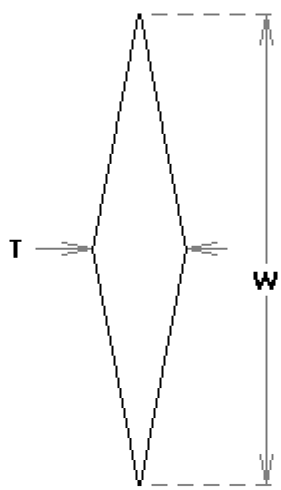
Some hold that it's a representation of the cross, a Christian symbol, placed there mostly for esthetics and religion. Unfortunately the Muslims also used it, as did pre-Christian Europeans. Even the name is odd, as in almost all other compound guard words the compounded word is the thing being guarded, like nose-guard, face-guard, border-guard, etc. However, we needn't pursue this idea too far, as the Oxford English Dictionary lists cross-guard as appearing in 1874. Maybe the same people who gave us the terms distal taper and broadsword, coined this word, too. The term "cross" was used earlier, as was the term "cross of a sword," which was seemingly limited to referring to praying to the cross of the sword, which is what you do when you place it point down, and use it like a cross. We also have the German term "cross piece". But many of the cross-guards don't look much like a cross at all. You see them swept forward, swelling at the ends, or twisted into delicate curved shapes at the guard's tips. You see all kinds of shapes, none seemingly optimized for anything in particular. If they were expressly designed to catch an opponent's blade they would all have some forward sweep, notches, or hooks. But most don't. If they were to stop the power of an opponent's blow, they would probably be thick near the blade, and taper toward the tips. But most get thick near the tips, not the blade. If they were optimized for use in a murder stroke, they would have pointy ends, or edged ends. But most don't. It seems that the cross-guards have nothing in common, except that they are always, always there. But almost never on axes, maces, or clubs. If mace technique is in any way like sword technique, even if just in similarity of combatant's training and opponents, any universal dependence on a cross-guard for defense should also carry over to a mace or club, which was used by the same peoples in the same battles. But this doesn't seem to happen.

Other mysteries are why Scotsmen would saw off the cross guards on their basket hilt claymores, which were imported from Spain. I've never heard of anyone in this day and age buying a well made sword, and then sawing off the cross guard. This implies that the Scotsmen didn't think it was needed on that type of sword. And of course, it's not needed. It has a basket hilt! But why did the Spanish think it was needed, and thus add it in the first place? Why did cross guards go from being small affairs, sufficient to prevent an edge from slipping into your fingers, to the gigantic guards seen on true two-handed swords? And why are they so often thickened on the end with a ball or other contrivance, such as with the earlier Scottish claymore? *These guards seemed to get larger at around the same period that knights started wearing maille and plate gauntlets on their hands.* Why would the addition of serious hand protection also come about with the use of a bigger guard, not a smaller one? With gauntlets on, why would anyone even worry about a blade touching their hands, when in a bind? Why, when the gauntlets finally disappeared (hard to reload a gun with those on), did the cross guard also see its demise? Strange mysteries indeed!

What if the cross-guard was doing something else entirely? What if it served to keep the sword from rotating around its long axis, when struck nearly on the flat during high-energy deflections and counter-cuts? What if it were also serving as a blade spin stabilizer? Sounds crazy? Well, until very recently it seems that most everyone was blocking with edges, not flats. Edge impacts don't have a moment arm sufficient to provide much torque to rotate the blade, so as to spin the grip around in your hands. Also, has anyone been doing these high-energy deflections with the flat, yet without a cross guard? The way our reproduction swords are constructed, the grip won't tighten up unless forced against the cross guard, so it's doubtful if anyone has tried using a sword that doesn't have a somewhat accurate reproduction cross guard in place. And finally, if you start putting thick gloves on, especially ones with metal on the palm side, your gripping force and coefficient of friction are dramatically reduced. Sometimes, seemingly to the point of trying to hold on to a greased handle.

The problem comes from our old friend, the moment of inertia. Except this time it's calculated about the long axis of the sword. Without a cross-guard, the moment of inertia of a straight medieval sword is very, very tiny. The cross guard likely makes up 95% or more of the completed sword's moment of inertia about this axis. The pommel and cross guard, taken together, probably account for more than 99% of the moment of inertia.

The rotational moment of inertia of diamond cross-section is calculated with the following formulas.



Mass Moment of Inertia of a Diamond

$$I_W = \frac{m W^2}{24}$$

MOI when rolled away from you, like a wheel, about an axel running through the middle

$$I_T = \frac{m T^2}{24}$$

MOI when spun with a vertical axis, like a top.

$$I_P = I_W + I_T$$

MOI when rotated about the COM in either the CCW or CW direction.

Given a sword with a 3/16″ blade thickness, a 1-¾″ blade width, and a mass of 1 kg, the moment of inertia about the spin axis is 0.00008327 kg-m^2. This is about 1500 times less than the sword's moment of inertia about the center of mass, when rotated to strike a target. When we factor in distal taper, this amount decreases by about half. One example of a taper from 0.25″ to 0.1″ in thickness, and 1-3/4″ to ¾″ in width, gave a blade mass of 1.29 lbs (0.585 kg) and a moment of inertia of just 0.0000422 kg-m^2. A simple cross of ½″ by ¼″ steel, 8″ (20 cm) in span, has a moment of inertia about its center that is over 10 times this amount. But if the cross is cut down to 6 inches, then it only has 4.4 times the moment of inertia of the blade. If made to span 10 inches, it provides 20 times more moment of inertia than the bare blade. Since the moment of inertia follows a square law property, the damping effect of a cross is very sensitive to its length. So make it as long as necessary and practical, so it can damp spin without interfering with the arms, and concentrate the mass at its ends if the practical length leaves the moment of inertia a bit short of desire.

When it comes to having some inertia to resist spinning, the bare blade just doesn't have anything there. Yet when you strike at your opponent and he violently redirects you blade, knocking it out of his way, the torque imparted about the spin axis can be very large. With that much torque, and no moment of inertia to resist it, the blade will reorient itself in milliseconds. Instead of having your edge aligned with your strike, it will suddenly shift to some large angle, which you can't control. You follow up strike will likely have the blade striking obliquely, 30 or even 45 degrees from straight. Obviously, this is completely unacceptable behavior.

This section will have to wait for more results. I tried gripping my 1840's NCO sword, which has a brass hilt, using maille and found that it doesn't work at all. I can't maintain blade orientation even against light taps with a bayonet! I had to tighten an adjustable wrench to the blade to damp the impact induced spin. The grip is so bad that surely no one could use mail gauntlets on it. I need to find out more about the exact palm materials used with mail, gauntlets, and any other types of hand protection. I also need to know what data there is in regards to which types of hand protections went with which types of grips. The friction of the iron maille on the brass grip is pretty much nonexistent, compared to the bare hand grip, so therein may lie the problem

Chapter 14
Measuring the Parameters

Fortunately these parameters are fairly easily measured, not quite as conveniently as weight, but still not very difficult. The units for moment of inertia are in kg-m^2 or lb-ft^2, mass multiplied by length squared. You can buy a piece of lab equipment to measure moment of inertia, but the only unit I found (http://www.idicb.com/moimeas.htm#options) cost $9,893 to $10,393, depending on options, and is accurate to only 0.5%. The other methods only require a stopwatch and either a weigh scale or piece of wire, so let's save some money and go with the simple methods.

The gravity pendulum method

1) Weigh your sword on an accurate scale and write down the mass (m) in kg or pounds.
2) Suspend your sword like a pendulum from some convenient point on it, like the cross guard or pommel. I like to place one end of the cross guard on a bookcase and hold the other end on my thumb. Using a stopwatch, time how long it takes for one complete swing out and back (period τ) just as if it were a pendulum in a grandfather clock. I usually time ten swings and divide the result by ten to decrease the error inherent in starting and stopping the stopwatch.
3) Measure the distance from where you suspended the sword (the axis of rotation) to the swords balance point. Call this distance x_S , the distance from suspension to center of mass.
4) Make sure all your units are in feet and pounds, or in kg and meters.
5) The percussion point location relative to the axis of suspension used in step 2 is given by the equation $Q_O = \frac{g \cdot \tau^2}{4 \cdot \pi^2}$, where g is either 9.8 m/sec^2 or 32.2 ft/sec^2. If you're using metric then Q_O is in meters. If you're using the English system then Q_O is in feet. If you've swung the sword from the blade side of the cross guard this gives you the percussion point from the bottom of your index finger if you finger the ricasso. Keep in mind that this is not the percussion point distance from the center of mass. This is from the axis of rotation, in my case the cross-guard.
6) The sword's moment of inertia about the axis of suspension used above is given by the equation $I_S = \frac{m \cdot x_S \cdot g \cdot \tau^2}{4 \cdot \pi^2}$, where g is either 9.8 m/sec^2 or 32.2 ft/sec^2, and the inertia is in kg-m^2 or lb-ft^2. All we're really doing is taking $I_S = m \cdot x_S \cdot Q_O$, so completing step 5 just leaves fewer numbers to crunch in step 6, while giving an initial position of the percussion point relative to the wrong side of the cross guard.
7) The swords moment of inertia about its own balance point is found by subtracting the swords mass times the distance x squared from the moment of inertia just calculated in step 6, so using the parallel axis theorem we have $I_{COM} = I_S - m \cdot x_S^2$.
8) Measure the distance from the swords balance point to the center of the grip, or where you think the center of your hand normally is (or between your hands for a two handed sword). Call this distance x .

9) The sword's moment of inertia about the grip is found by adding to the sword's moment of inertia about its balance point (I_{COM} calculated in step 5) the mass times distance x squared, giving $I_X = I_{COM} + m \cdot x^2$.

10) Now you know the moment of inertia about the center of mass, I_{COM}, and the mass m. From the center of mass (hoped you marked it!) measure off the distance to your top hand's first knuckle. Then use a pocket calculator to compute $Q = \frac{I_{COM}}{m \cdot x}$. Measure off this calculated distance, again from the center of mass, and mark it on your blade. Repeat this for your pinky knuckle and you have a good idea of the range of the blade's impact locations that result in small hand shock.

The Torsion Pendulum Method

This method of measuring your sword's moment of inertia is given in equations 9 and 10 found here, http://www.colorado.edu/physics/phys1140/phys1140_sm98/Experiments/M4/M4.html, along with the theory of the torsion pendulum. I won't go into it except to say that your wire should be spring steel, like music wire, dead straight, and suspended so that it can't slip anywhere. Get several different pieces of steel rod from the hardware store to calibrate your rig and verify it across a range of weights and moments of inertia. The equation for a long rod about its center is $I_{COM} = \frac{m \cdot L^2}{12}$.

Work with your torsion pendulum till it gives you correct results on the rods, and then measure the swords.

The textbook formula for the moment of inertia of a straight, thin object about the object's center of mass is

$\mathbf{I_{COM}=mL^2/12}$

Where **m** is the object's mass and **L** is its length, from end to end.

To find the moment of inertia at one end you can look up the formula or use the parallel axis theorem.

$\mathbf{I=I_{COM} + mx^2}$

Where for an end **x** is ½ **L**, so

$\mathbf{I_{END}=I_{COM} + m(L/2)^2}$, or $\mathbf{I=I_{COM}+mL^2/4}$

Substituting in the formula for $\mathbf{I_{COM}}$

$\mathbf{I_{END}=mL^2/12 + mL^2/4}$

Doing a bit of algebra,

$\mathbf{I_{END}=mL^2/12 + 3mL^2/12}$

$\mathbf{I_{END}=4mL^2/12}$

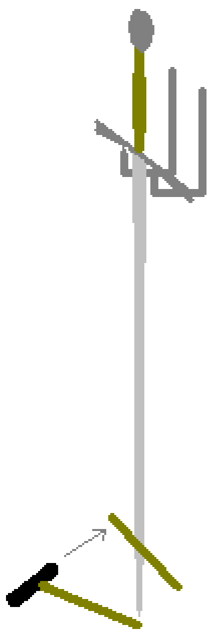
$\mathbf{I_{END}=mL^2/3}$

Which matches the textbook formula. The textbook formula for $\mathbf{I_{COM}}$ makes it handy to run to the hardware store and buy a piece of stock material for calibrating your torsion balance.

Impact Methods

There are a wide variety of ways to use a sharp impact to make the percussion point reveal itself. Once you know the mass, center of mass location, and percussion point location you can directly calculate the moment of inertia. But in comparing replica blades we've shown that as long as three parameters are correct then the fourth must also be correct.   So simply measuring the percussion point is sufficient for controlling the moment of inertia. And if the percussion point is the crucial parameter then you don't even need to worry about the moment of inertia, as long as your mass and balance point are near authentic values.

One method to find the percussion point is by hanging the sword vertically by balancing the cross-guard on a pair of nails or rods hangers, as illustrated. Gently place a small dowel against a measured point on the blade and then strike the dowel with a mallet. The cross- guard will kick off the nails toward the mallet if the dowel was out past the percussion point. If the cross-guard kicks of the nails away from the mallet the dowel was inside the percussion point. If the sword swings like a pendulum before falling off the nails then the dowel is very close to the percussion point.



This test can also be performed with the blade pointing upward and the hand side of the cross-guard resting on two vertical spikes. This has the advantage of not having the sword fall on you and making the strike a bit more convenient. It also finds the percussion point relative to the hand of the cross-guard, instead of the blade side.

Chapter 15
Further Research

This article explains a great deal, but also, as a consequence, brings up many things that we need to find out about. Here is a short list of additional research that we need to pursue.

- What is the moment of inertia of authentic swords, broken down by style, period, etc?
- What is the statistical relation between MOI, mass, length, balance point, and percussion point on authentic swords?
- Given these relations, how tightly are medieval swords clustered, in regard to these parameters.
- Is the distance from the tip to the percussion point tightly controlled, across and within each type?
- In swords that are optimized for thrusting, is the MOI allowed to grow larger than normal, based on the statistics collected for the previous questions.
- As the percussion point is irrelevant in thrusts, do pure thrusting weapons have controlled percussion points?
- Is there a relation between percussion point location and the area of the tip that is sharpened?
- Is there any evidence in the statistics collected above, that the swordsmen's preferred percussion point locations varied over different time periods?
- Does the moment of inertia of surviving swords vary evenly across and among types, or is it clustered, revealing different styles of combat or different weapon types?
- If the percussion point location varies by a fairly significant margin, do the swords with locations closer to the tip also incorporate stiffer tip designs, edge-to-edge?
- If the percussion point locations are tightly clustered into two slightly different groups, are there any corresponding hilt features that also correlate the same. For example, would the difference in the percussion point locations, divide the swords into those that were probably fingered at the cross-guard, versus those that were not?
- Given sets of swords whose grips have a similar coefficient of friction, and whose blades are of similar width, is there a correlation with the moment of inertia of the cross-guard, when rotated about the long axis of the sword.
- Is there a correlation between moment of inertia about the swords long axis, or the spin axis, and features of the grip, such as material, coefficient of friction, diameter, and single or double handedness?
- Is there a correlation between moment of inertia about the spin axis and blade stiffness, as stiff blades don't flex as much, so they don't damp spin as well.
- What are the typical stiffness profiles of medieval blades?
- Is there a correlation between tang strength, and the torque that an arbitrary impact at the forefinger's percussion point would apply to the pommel?
- What is the typical ratio of tang strength and stiffness, in the edge-to-edge versus the side-to-side planes?
- Is the entire tang soft, which would likely cause it to take permanent deformations, or is mainly tempered spring steel with only a soft tip, where it gets peened to hold the pommel?
- What are the vibration characteristics of ordinary swords? What is the normal frequency range of fundamental, second, and third harmonic oscillations in the edge-to-edge plane?
- What is the stiffness profile on medieval swords? Hanging a weight on the tip, and then carefully recording the resulting displacement of the entire blade, can find this. From the displacement it's possible to calculate the corresponding radius of curvature, and the stiffness profile can be plotted from that data.

- What is the stiffness profile, edge-to-edge, of the different sword types? This is probably best approached by measuring the cross-sections of medieval blades (standard production equipment can be used to perform the surface profiling, and dump the data into a computer), and then using mechanical engineering software to back-calculate the resulting stiffness. If not, the formulas are simple, and the job could be performed by hand, though the data collection and analysis would be laborious.
- Given a variety of blade edges, what is the function of impact distance to blade-obliquity, in a variety of materials? Basically, if your blade isn't aligned with the impact, how much does it matter?
- What is the curve of impact distance versus the amount that a given cut is drawn back, as a function of the draw angle? How much draw makes a draw cut work?
- How is the benefit of drawing a cut, affected by the target material? Obviously its futile on steel, but of some benefit on flesh. What about leather, mail, linen, etc?
- What is the effect of edge thickness angle on penetration, again in a variety of materials?
- How is the maximum force deliverable in a thrust, a function of blade stiffness, end to end?
-